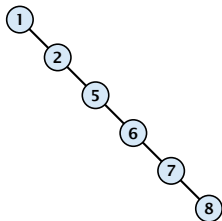
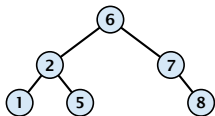


## 7.1 Binary Search Trees

An (**internal**) **binary search tree** stores the elements in a binary tree. Each tree-node corresponds to an element. All elements in the left sub-tree of a node  $v$  have a smaller key-value than  $\text{key}[v]$  and elements in the right sub-tree have a larger-key value. We assume that all key-values are different.

(**External** Search Trees store objects only at leaf-vertices)

Examples:



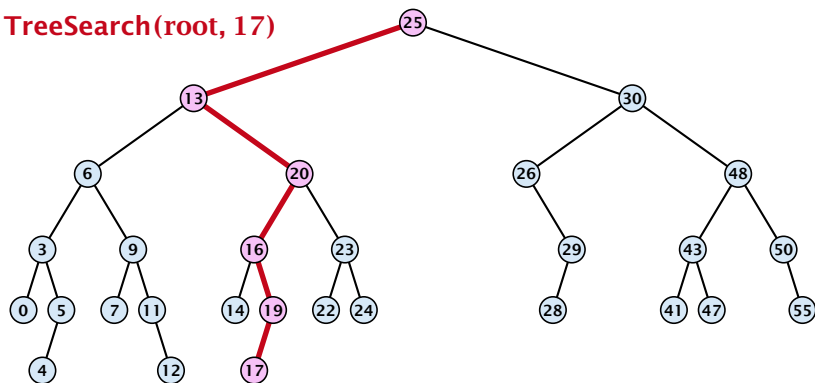
## 7.1 Binary Search Trees

We consider the following operations on binary search trees. Note that this is a super-set of the dictionary-operations.

- ▶  $T.\text{insert}(x)$
- ▶  $T.\text{delete}(x)$
- ▶  $T.\text{search}(k)$
- ▶  $T.\text{successor}(x)$
- ▶  $T.\text{predecessor}(x)$
- ▶  $T.\text{minimum}()$
- ▶  $T.\text{maximum}()$

# Binary Search Trees: Searching

TreeSearch(root, 17)

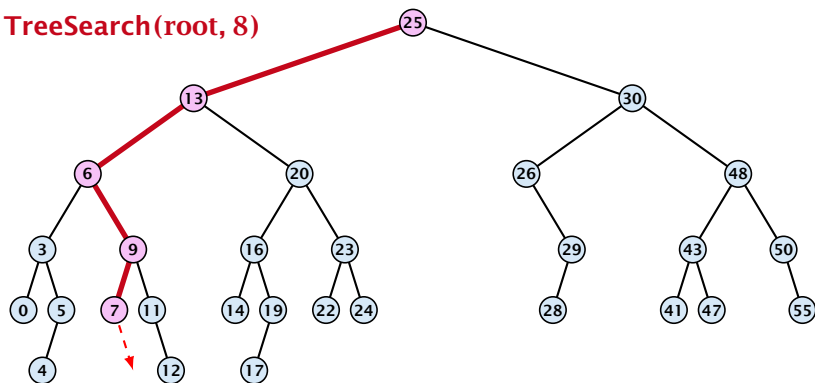


## Algorithm 1 TreeSearch( $x, k$ )

- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
- 2: **if**  $k < \text{key}[x]$  **return** TreeSearch(left[ $x$ ],  $k$ )
- 3: **else return** TreeSearch(right[ $x$ ],  $k$ )

# Binary Search Trees: Searching

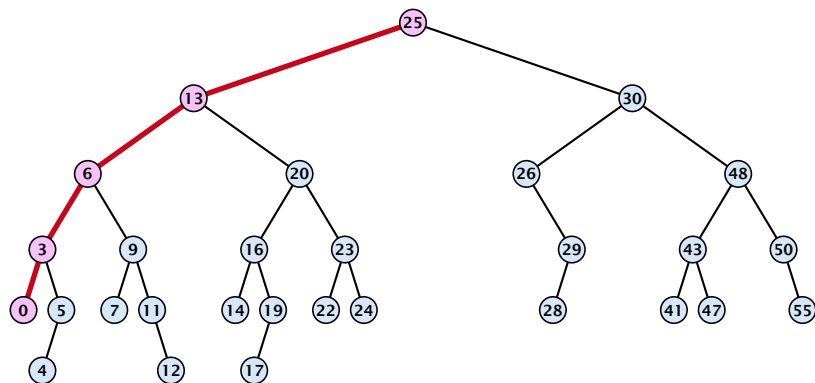
TreeSearch(root, 8)



## Algorithm 1 TreeSearch( $x, k$ )

- 1: **if**  $x = \text{null}$  **or**  $k = \text{key}[x]$  **return**  $x$
- 2: **if**  $k < \text{key}[x]$  **return** TreeSearch(left[ $x$ ],  $k$ )
- 3: **else return** TreeSearch(right[ $x$ ],  $k$ )

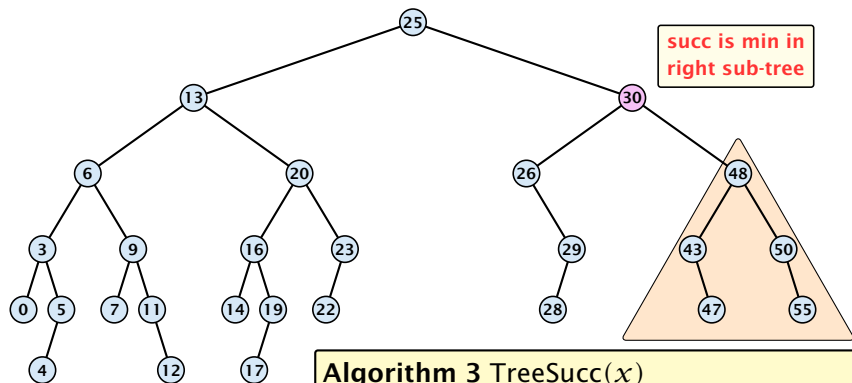
# Binary Search Trees: Minimum



## Algorithm 2 TreeMin( $x$ )

- 1: if  $x = \text{null}$  or  $\text{left}[x] = \text{null}$  return  $x$
- 2: return TreeMin( $\text{left}[x]$ )

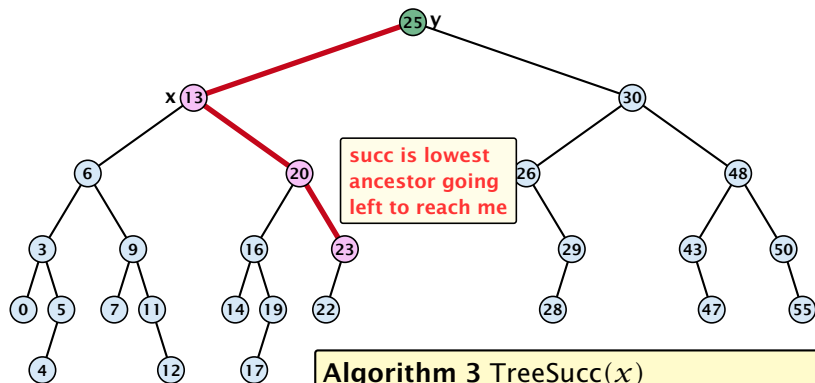
# Binary Search Trees: Successor



## Algorithm 3 TreeSucc( $x$ )

- 1: **if** right[ $x$ ]  $\neq$  null **return** TreeMin(right[ $x$ ])
- 2:  $y \leftarrow$  parent[ $x$ ]
- 3: **while**  $y \neq$  null **and**  $x =$  right[ $y$ ] **do**
- 4:      $x \leftarrow y$ ;  $y \leftarrow$  parent[ $x$ ]
- 5: **return**  $y$ ;

# Binary Search Trees: Successor



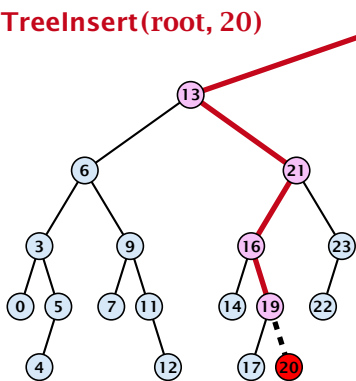
## Algorithm 3 TreeSucc( $x$ )

- 1: **if** right[ $x$ ]  $\neq$  null **return** TreeMin(right[ $x$ ])
- 2:  $y \leftarrow$  parent[ $x$ ]
- 3: **while**  $y \neq$  null **and**  $x =$  right[ $y$ ] **do**
- 4:      $x \leftarrow y$ ;  $y \leftarrow$  parent[ $x$ ]
- 5: **return**  $y$ ;

## Binary Search Trees: Insert

Insert element **not** in the tree.

**TreeInsert**(root, 20)



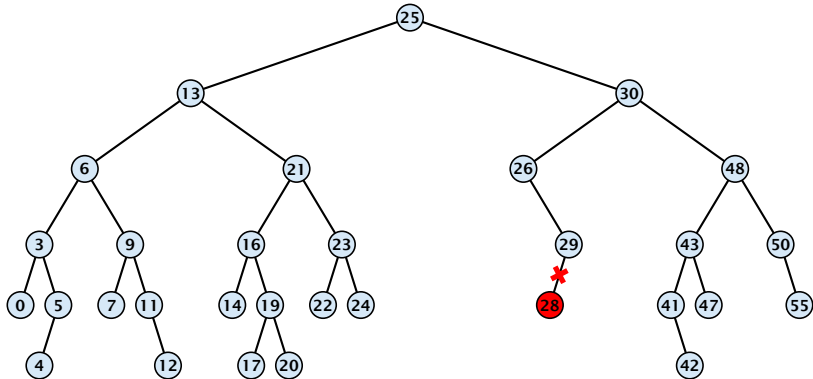
Search for  $z$ . At some point the search stops at a null-pointer. This is the place to insert  $z$ .

### Algorithm 4 TreeInsert( $x, z$ )

```
1: if  $x = \text{null}$  then
2:    $\text{root}[T] \leftarrow z$ ;  $\text{parent}[z] \leftarrow \text{null}$ ;
3:   return;
4: if  $\text{key}[x] > \text{key}[z]$  then
5:   if  $\text{left}[x] = \text{null}$  then
6:      $\text{left}[x] \leftarrow z$ ;  $\text{parent}[z] \leftarrow x$ ;
7:   else TreeInsert( $\text{left}[x], z$ );
8: else
9:   if  $\text{right}[x] = \text{null}$  then
10:     $\text{right}[x] \leftarrow z$ ;  $\text{parent}[z] \leftarrow x$ ;
11:   else TreeInsert( $\text{right}[x], z$ );
```



# Binary Search Trees: Delete

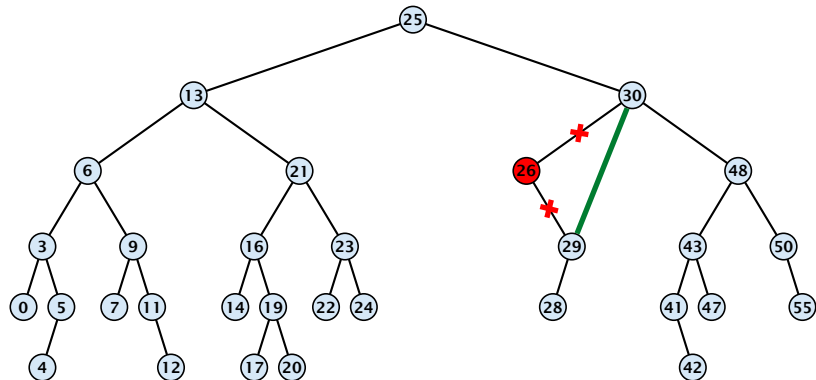


## Case 1:

Element does not have any children

- ▶ Simply go to the parent and set the corresponding pointer to **null**.

# Binary Search Trees: Delete

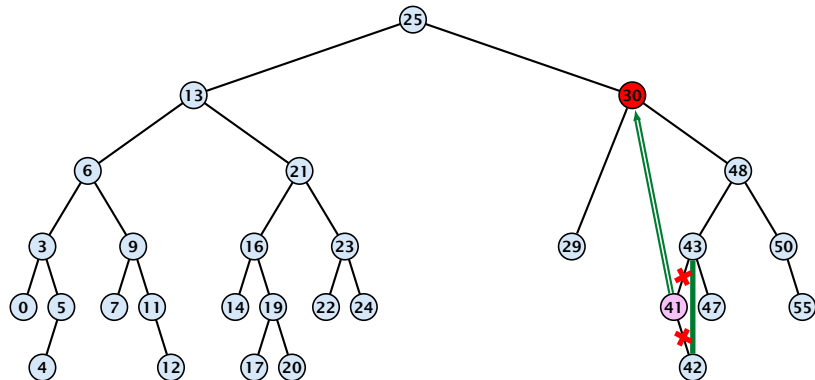


## Case 2:

Element has exactly one child

- ▶ Splice the element out of the tree by connecting its parent to its successor.

# Binary Search Trees: Delete



## Case 3:

Element has two children

- ▶ Find the successor of the element
- ▶ Splice successor out of the tree
- ▶ Replace content of element by content of successor

# Binary Search Trees: Delete

## Algorithm 9 TreeDelete( $z$ )

```
1: if left[ $z$ ] = null or right[ $z$ ] = null
2:   then  $y \leftarrow z$  else  $y \leftarrow \text{TreeSucc}(z)$ ;   select  $y$  to splice out
3:   if left[ $y$ ]  $\neq$  null
4:     then  $x \leftarrow \text{left}[y]$  else  $x \leftarrow \text{right}[y]$ ;  $x$  is child of  $y$  (or null)
5:     if  $x \neq \text{null}$  then parent[ $x$ ]  $\leftarrow$  parent[ $y$ ];   parent[ $x$ ] is correct
6:     if parent[ $y$ ] = null then
7:       root[ $T$ ]  $\leftarrow x$ 
8:     else
9:       if  $y = \text{left}[\text{parent}[y]]$  then
10:        left[parent[ $y$ ]]  $\leftarrow x$ 
11:      else
12:        right[parent[ $y$ ]]  $\leftarrow x$ 
13:   if  $y \neq z$  then copy  $y$ -data to  $z$ 
```

} fix pointer to  $x$

# Balanced Binary Search Trees

All operations on a binary search tree can be performed in time  $\mathcal{O}(h)$ , where  $h$  denotes the height of the tree.

However the height of the tree may become as large as  $\Theta(n)$ .

## Balanced Binary Search Trees

With each insert- and delete-operation perform **local** adjustments to guarantee a height of  $\mathcal{O}(\log n)$ .

AVL-trees, Red-black trees, Scapegoat trees, 2-3 trees, B-trees, AA trees, Treaps

similar: SPLAY trees.

# Binary Search Trees (BSTs)

## Bibliography

- [MS08] Kurt Mehlhorn, Peter Sanders:  
*Algorithms and Data Structures — The Basic Toolbox*,  
Springer, 2008
- [CLRS90] Thomas H. Cormen, Charles E. Leiserson, Ron L. Rivest, Clifford Stein:  
*Introduction to Algorithms (3rd ed.)*,  
MIT Press and McGraw-Hill, 2009

Binary search trees can be found in every standard text book. For example Chapter 7.1 in [MS08] and Chapter 12 in [CLRS90].