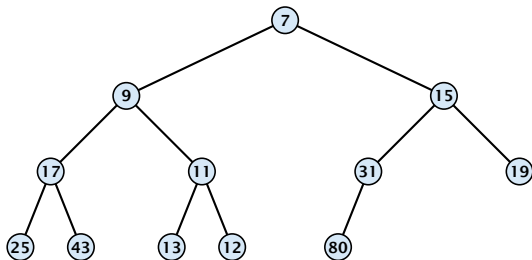
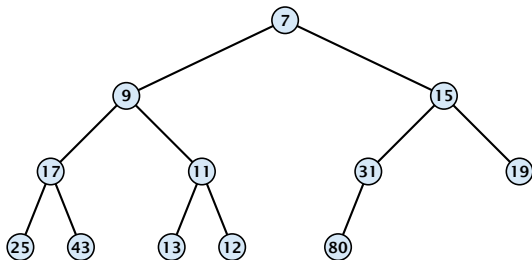


## 8.1 Binary Heaps



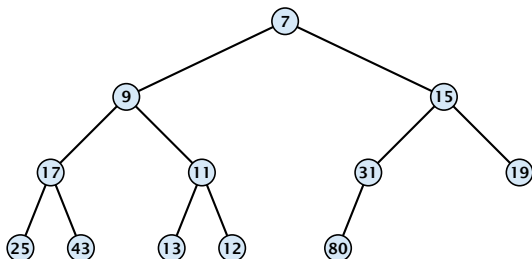
## 8.1 Binary Heaps

- ▶ Nearly complete binary tree; only the last level is not full, and this one is filled from left to right.



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- ▶ Nearly complete binary tree; only the last level is not full, and this one is filled from left to right.
- ▶ **Heap property:** A node's key is not larger than the key of one of its children.



**Operations:**

# Binary Heaps

## Operations:

- ▶ **minimum()**: return the root-element. Time  $\mathcal{O}(1)$ .

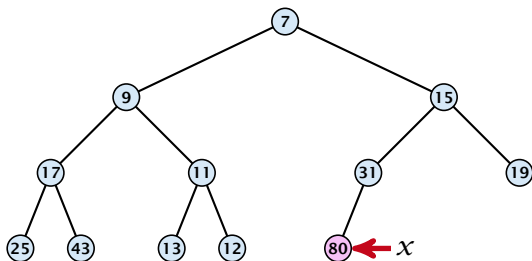
# Binary Heaps

## Operations:

- ▶ **minimum()**: return the root-element. Time  $\mathcal{O}(1)$ .
- ▶ **is-empty()**: check whether root-pointer is **null**. Time  $\mathcal{O}(1)$ .

## 8.1 Binary Heaps

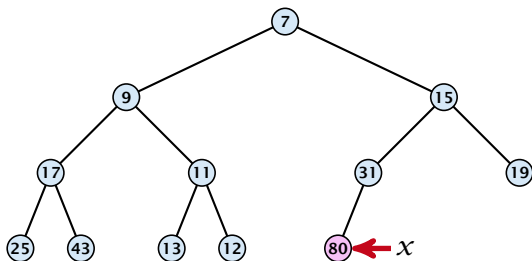
Maintain a pointer to the **last element**  $x$ .



## 8.1 Binary Heaps

Maintain a pointer to the **last element**  $x$ .

- ▶ We can compute the predecessor of  $x$  (last element when  $x$  is deleted) in time  $\mathcal{O}(\log n)$ .





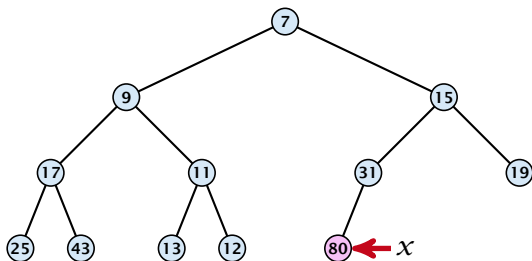
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go up until the last edge used was a right edge.

go left; go right until you reach a leaf



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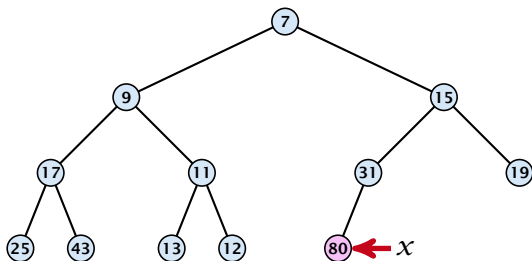
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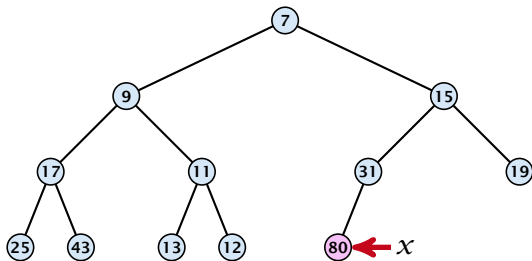
go left; go right until you reach a leaf

if you hit the root on the way up, go to the rightmost element



## 8.1 Binary Heaps

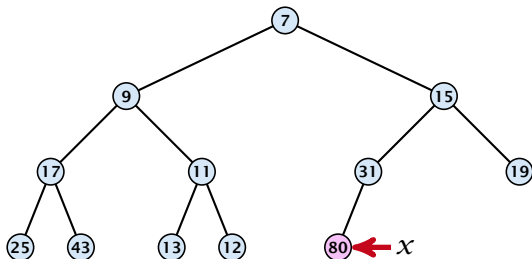
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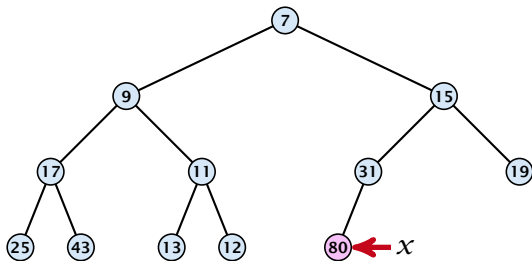
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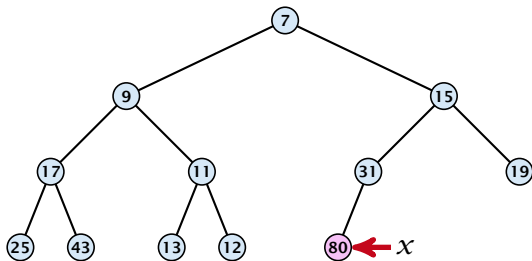
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go up until the last edge used was a left edge.

go right; go left until you reach a **null**-pointer.

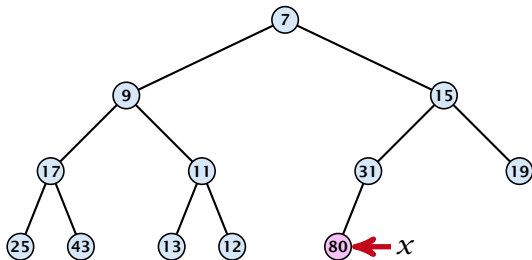
if you hit the root on the way up, go to the leftmost element;

insert a new element as a left child;



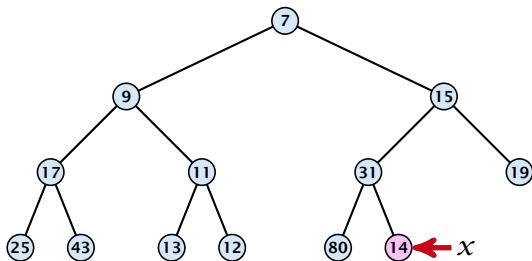
# Insert

1. Insert element at successor of  $x$ .



# Insert

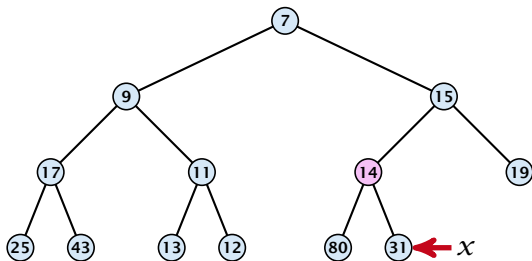
1. Insert element at successor of  $x$ .
2. Exchange with parent until heap property is fulfilled.





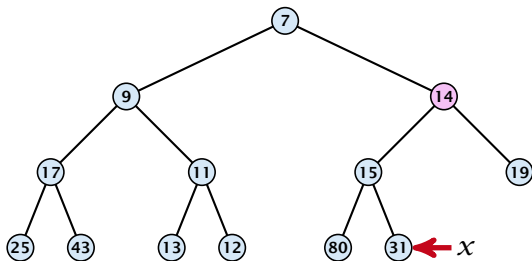
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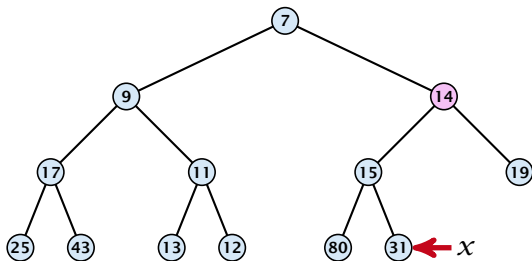
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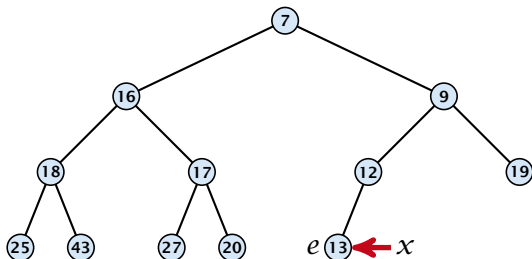
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Note that an exchange can either be done by moving the data or by changing pointers. The latter method leads to an addressable priority queue.

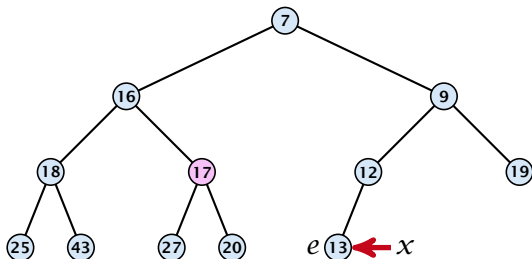
# Delete

1. Exchange the element to be deleted with the element  $e$  pointed to by  $x$ .



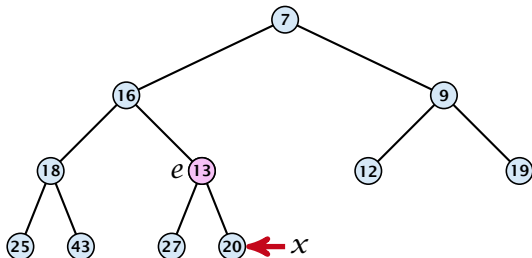
# Delete

1. Exchange the element to be deleted with the element  $e$  pointed to by  $x$ .
2. Restore the heap-property for the element  $e$ .



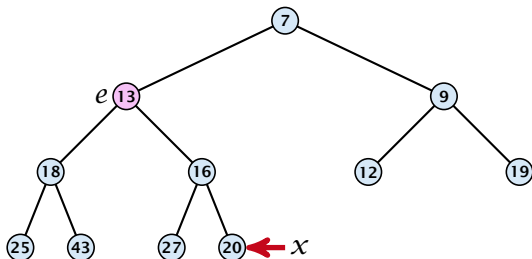
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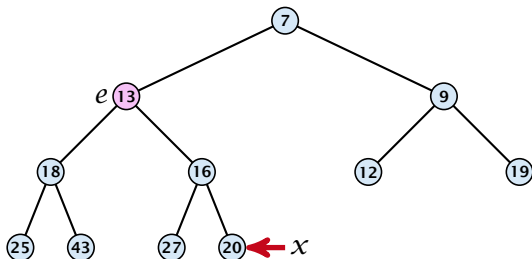
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At its new position  $e$  may either travel up or down in the tree (but not both directions).



# Binary Heaps

## Operations:

- ▶ **minimum()**: return the root-element. Time  $\mathcal{O}(1)$ .
- ▶ **is-empty()**: check whether root-pointer is **null**. Time  $\mathcal{O}(1)$ .
- ▶ **insert( $k$ )**: insert at successor of  $x$  and bubble up. Time  $\mathcal{O}(\log n)$ .
- ▶ **delete( $h$ )**: swap with  $x$  and bubble up or sift-down. Time  $\mathcal{O}(\log n)$ .

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- ▶ **delete( $h$ )**: Swap with  $x$  and bubble up or sift-down. Time  $\mathcal{O}(\log n)$ .
- ▶ **build( $x_1, \dots, x_n$ )**: Insert elements arbitrarily; then do sift-down operations starting with the lowest layer in the tree. Time  $\mathcal{O}(n)$ .

# Binary Heaps

# Binary Heaps

The standard implementation of binary heaps is via arrays. Let  $A[0, \dots, n - 1]$  be an array

- ▶ The parent of  $i$ -th element is at position  $\lfloor \frac{i-1}{2} \rfloor$ .
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The resulting binary heap is not addressable. The elements don't maintain their positions and therefore there are no stable handles.