WS 2019/20

Efficient Algorithms and Data Structures

Harald Räcke

Fakultät für Informatik
TU München

http://www14.in.tum.de/lehre/2019WS/ea/

Winter Term 2019/20

Part I

Organizational Matters

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- Modul: IN2003
- Name: "Efficient Algorithms and Data Structures"
  "Effiziente Algorithmen und Datenstrukturen"
- ECTS: 8 Credit points
- Lectures:
  - 4 SWS
    Mon 10:00–12:00 (Room Interim2)
    Fri 10:00–12:00 (Room Interim2)
- Webpage: `http://www14.in.tum.de/lehre/2019WS/ea/`

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    “Fundamentals of Algorithms and Data Structures”
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The Lecturer

- Harald Räcke
- Email: raecke@in.tum.de
- Room: 03.09.044
- Office hours: (by appointment)

Tutorials

A01 Monday, 12:00–14:00, 00.08.038 (Stotz)

A02 Monday, 12:00–14:00, 00.09.038 (Guan)

A03 Monday, 14:00–16:00, 02.09.023 (Stotz)

B04 Tuesday, 10:00–12:00, 00.08.053 (Czerner)

B05 Tuesday, 14:00–16:00, 00.08.038 (Czerner)

C06 Wednesday, 10:00–12:00, 03.11.018 (Guan)

E07 Friday, 12:00–14:00, 00.13.009 (Stotz)

Assignment sheets

In order to pass the module you need to pass an exam.

Assessment

Assignment Sheets:

- An assignment sheet is usually made available on Monday on the module webpage.
- Solutions have to be handed in in the following week before the lecture on Monday.
- You can hand in your solutions by putting them in the mailbox "Efficient Algorithms" on the basement floor in the MI-building.
- Solutions have to be given in English.
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- You should submit solutions in groups of up to 2 people.

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Assessment

Assignment Sheets:

- Submissions must be handwritten by a member of the group. Please indicate who wrote the submission.
- Don't forget name and student id number for each group member.

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Assessment

Assignment can be used to improve you grade

Requirements for Bonus

- 50% of the points are achieved on submissions 2–8,
- 50% of the points are achieved on submissions 9–14,
- each group member has written at least 4 solutions.

1 Contents

- Foundations
  - Machine models
  - Efficiency measures
  - Asymptotic notation
  - Recursion
- Higher Data Structures
  - Search trees
  - Hashing
  - Priority queues
  - Union/Find data structures
- Cuts/Flows
- Matchings

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2 Literatur

- Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman:
  The design and analysis of computer algorithms,
  Addison-Wesley Publishing Company: Reading (MA), 1974
- Thomas H. Cormen, Charles E. Leiserson, Ron L. Rivest, Clifford Stein:
  Introduction to algorithms,
  McGraw-Hill, 1990
- Michael T. Goodrich, Roberto Tamassia:
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  John Wiley & Sons, 2002

2 Literatur

- Ronald L. Graham, Donald E. Knuth, Oren Patashnik:
  Concrete Mathematics,
  2. Auflage, Addison-Wesley, 1994
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  Grundlegende Algorithmen: Einführung in den Entwurf und die Analyse effizienter Algorithmen,
  2. Auflage, Vieweg, 2003
- Jon Kleinberg, Eva Tardos:
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- Donald E. Knuth:
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Christos H. Papadimitriou, Kenneth Steiglitz:
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Prentice Hall, 1982

Uwe Schöning:
Algorithmik,
Spektrum Akademischer Verlag, 2001

Steven S. Skiena:
The Algorithm Design Manual,
Springer, 1998

Part II

Foundations

3 Goals

- Gain knowledge about efficient algorithms for important problems, i.e., learn how to solve certain types of problems efficiently.
- Learn how to analyze and judge the efficiency of algorithms.
- Learn how to design efficient algorithms.

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(2)

Input $\longrightarrow$ Output

I $\qquad$ $\mathcal{O}$

$f_A : I \to \mathcal{O}$

- A holds on every $x \in I$
- $f_A = f_P$

Problem : $f_P : I \to \mathcal{O}$

4 Modelling Issues

What do you measure?

- Memory requirement
- Running time
- Number of comparisons
- Number of multiplications
- Number of hard-disc accesses
- Program size
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4 Modelling Issues

How do you measure?

- Implementing and testing on representative inputs
  - How do you choose your inputs?
  - May be very time-consuming.
  - Very reliable results if done correctly.
  - Results only hold for a specific machine and for a specific set of inputs.
- Theoretical analysis in a specific model of computation.
  - Gives asymptotic bounds like “this algorithm always runs in time $\mathcal{O}(n^2)$”.
  - Typically focuses on the worst case.
  - Can give lower bounds like “any comparison-based sorting algorithm needs at least $\Omega(n \log n)$ comparisons in the worst case”.

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4 Modelling Issues

Input length

The theoretical bounds are usually given by a function $f : \mathbb{N} \to \mathbb{N}$ that maps the input length to the running time (or storage space, comparisons, multiplications, program size etc.).

The input length may e.g. be

- the size of the input (number of bits)
- the number of arguments

Suppose n numbers from the interval $\{1, \dots, N\}$ have to be sorted. In this case we usually say that the input length is n instead of e.g. $n \log N$, which would be the number of bits required to encode the input.

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Model of Computation

How to measure performance

- Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM), . . .
- Calculate number of certain basic operations: comparisons, multiplications, harddisc accesses, . . .

Version is often easier, but focusing on one type of operation makes it more difficult to obtain meaningful results.

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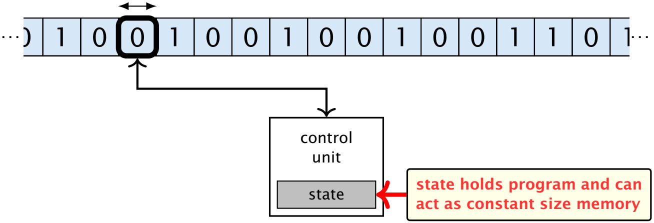
Turing Machine

- Very simple model of computation.
- Only the “current” memory location can be altered.
- Very good model for discussing computabiliy, or polynomial vs. exponential time.
- Some simple problems like recognizing whether input is of the form xx, where x is a string, have quadratic lower bound.

$\Rightarrow$ **Not a good model for developing efficient algorithms.**



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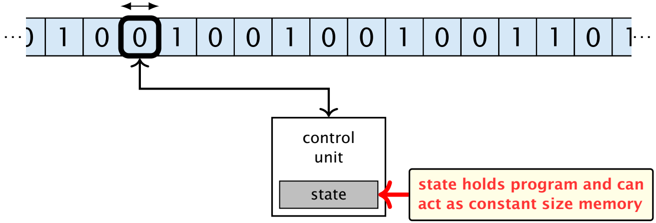

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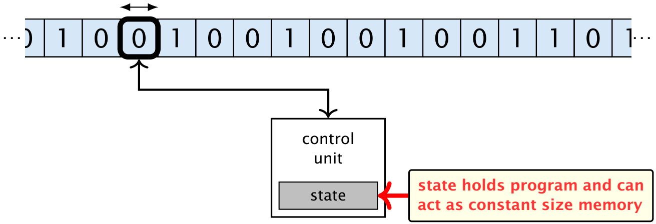

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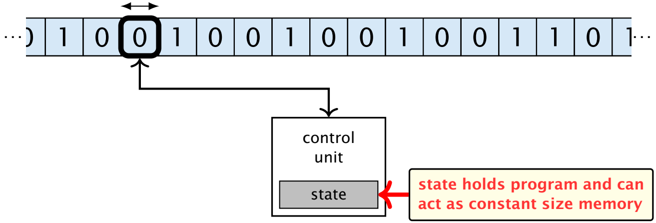

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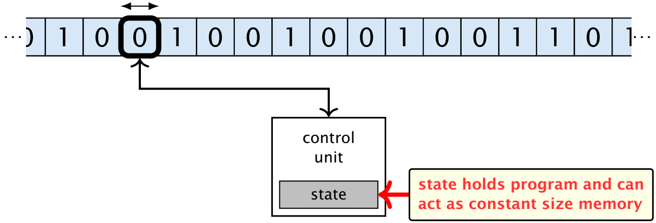
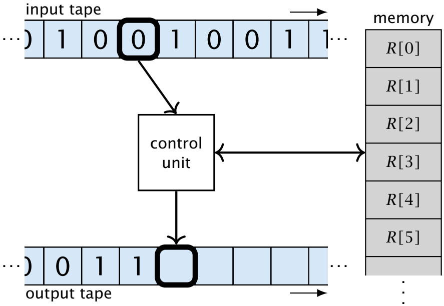


Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
- Indirect addressing.



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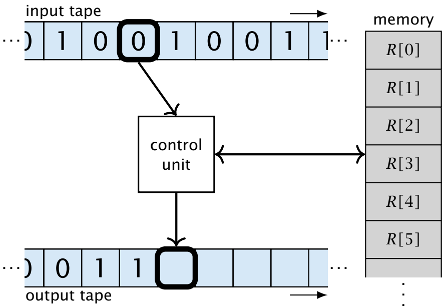

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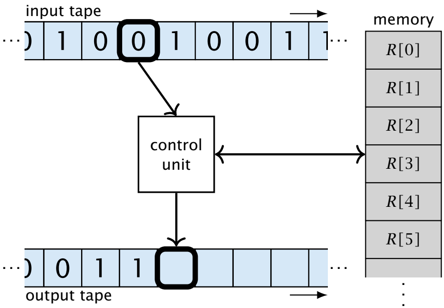

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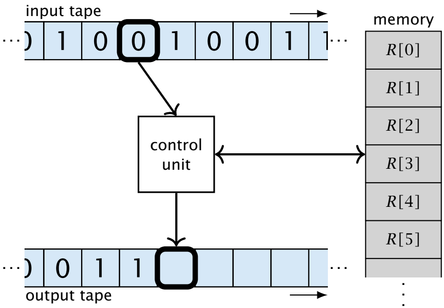

Random Access Machine (RAM)

Operations

- input operations (input tape → $R[i]$)
  - READ i
- output operations ($R[i]$ → output tape)
  - WRITE i
- register-register transfers
  - $R[j] := R[i]$
  - $R[j] := 4$
- **indirect** addressing
  - $R[j] := R[R[i]]$
    loads the content of the $R[i]$-th register into the j-th register
  - $R[R[i]] := R[j]$
    loads the content of the j-th into the $R[i]$-th register

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  - READ i
- output operations ($R[i]$ → output tape)
  - WRITE i
- register-register transfers
  - $R[j] := R[i]$
  - $R[j] := 4$
- **indirect** addressing
  - $R[j] := R[R[i]]$
    loads the content of the $R[i]$-th register into the j-th register
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  - `jumpz` x $R[i]$
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  Every operation takes time 1.
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  The cost depends on the content of memory cells:
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Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed 2^w, where usually $w = \log_2 n$.

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Algorithm 1 RepeatedSquaring(n)

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r ← 2;
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- best-case complexity:

$$C_{bc}(n) := \min\{C(x) \mid |x| = n\}$$

  Usually easy to analyze, but not very meaningful.

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↑ input length

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