Efficient Algorithms and Data Structures II

*Deadline: July 22, 2019, 10:15 am in the **Efficient Algorithms** folder.*

Homework 1 (5 Points)

In this exercise, we aim to show that MAX-3SAT(29) is NP-hard with a constant gap of ε.

Let 3SAT(k) be the 3SAT problem restricted to instances, in which each variable occurs at most k times. Similarly, let MAX-3SAT(k) be the MAX-3SAT problem restricted to instances, where each variable occurs at most k times.

1. Show that 3SAT(5) is NP-complete.

2. A graph G is an *expander* if every vertex has the same degree and for any subset $\emptyset \neq S \subset V$, we have

    $$\delta(S) > \min\{|S|, |V \setminus S|\} \;,$$

    where $\delta(S)$ denotes the set of edges leaving S.

    It is known that for each n larger than some constant N_0, a 14-regular n'-node expander can be constructed efficiently with $n' = n(1 + o(1))$.

    Use an expander to enhance the construction from Part 1 to prove the following theorem: There is a gap preserving reduction from MAX-3SAT to MAX-3SAT(29) that transforms a Boolean formula ϕ to ψ such that

    -   if $\text{OPT}(\phi) = m$, then $\text{OPT}(\psi) = m'$, and
    -   if $\text{OPT}(\phi) < (1 - \varepsilon)m$, then $\text{OPT}(\psi) < (1 - \varepsilon_b)m'$, where m and m' are the number of clauses in ϕ and ψ, resp., and $\varepsilon_b = \varepsilon/43$.

3. Show that a similar theorem holds for 3SAT-MAX(5).

Homework 2 (5 Points)

Have questions? Send them to the tutor ≥ 48 hours before the tutorial and win amazing prizes!

Tutorial Exercise 1

Prove the following claim: For any MAX-E3SAT formula ϕ, there exists an assignment satisfying at least $\frac{7}{8}m$ clauses. The MAX-E3SAT problem is a variant of MAX-3SAT where each clause of ϕ contains *exactly* 3 distinct variables.

Proof. A random truth assignment to the variables fulfills any clause with probability 7/8, hence the expected number of clauses we can fulfill is $7/8m$. The optimal number is larger than the expected one. $\qquad\square$

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Amusez-vous,
foutez-vous tout.
La vie, entre nous, est si brève.
Amusez-vous,
comme des fous
La vie est si courte après tout.
Car l'on est pas ici pour se faire du souci
on n'est pas ici bas pour se faire du tracas.
                          - A. Willemetz
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