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Summer term 2019
Assignment 8
June 17, 2019

Efficient Algorithms and Data Structures II

*Deadline: June 24, 2019, 10:15 am in the **Efficient Algorithms** folder.*

Homework 1 (5 Points)

Given an undirected graph $G = (V,E)$, a valid k-coloring is an assignment of its vertices to k colors such that the two endpoints of each edge receive distinct colors. The minimum vertex coloring problem is to find the minimum k such that G is k-colorable.

1. Give an algorithm for coloring G with $\Delta + 1$ colors, where Δ is the maximum degree of a vertex in G.
2. Give an algorithm for coloring a 3-colorable graph with $\mathcal{O}(\sqrt{n})$ colors.

Homework 2 (5 Points)

1. Prove that any *vertex* point of the LP

$$\begin{array}{llcll} \text{min.} & \sum_{v\in V} x_v & & & \\ \text{s.t.} & x_u + x_v & \geq & 1 & \{u,v\} \in E \\ & x_v & \geq & 0 & v \in V \ . \end{array}$$

   has the property that $x_v \in \{0, 1/2, 1\}$ for all $v \in V$.

2. Give a $\frac{3}{2}$-approximation algorithm for the vertex cover problem when the input graph is planar. Use the facts that we can find an optimal "vertex" point in polynomial time and there is a polynomial time algorithm to 4-color any planar graph.

Homework 3 (5 Points)

In the directed Steiner tree problem, we are given as input a directed graph $G = (V,E)$, nonnegative costs $c_{ij} \geq 0$ for edges $(i,j) \in E$, a root vertex $r \in V$, and a set of terminals $T \subseteq V$. The goal is to find a minimum-cost tree such that for each $i \in T$ there exists a directed path from r to i.
Prove that for some constant c there can be no $c\log|T|$-approximation algorithm for the directed Steiner tree problem, unless P = NP.
Hint: Use a reduction from the Set Cover Problem.

Bonus Homework 1

Given positive numbers $a_1,\ldots,a_k$ and $b_1,\ldots,b_k$ and $S \subseteq [k]$ show

$$\min_i \frac{a_i}{b_i} \leq \frac{\sum_{i\in S} a_i}{\sum_{i\in S} b_i} \leq \max_i \frac{a_i}{b_i} \ .$$

Tutorial Exercise 1

In the uncapacitated facility location problem, we have a set of clients D and a set of facilities F . For each client $j \in D$ and facility $i \in F$, there is a cost c_{ij} of assigning client j to facility i. Furthermore, there is a cost f_i associated with each facility $i \in F$. The goal of the problem is to choose a subset of facilities $F' \subseteq F$ so as to minimize the total cost of the facilities in F' and the cost of assigning each client $j \in D$ to the nearest facility in F' . In other words, we wish to find F' so as to minimize $\sum_{i \in F'} f_i + \sum_{j \in D} \min_{i \in F'} c_{ij}$.

1. Show that there exists some c such that there is no $(c \ln |D|)$-approximation algorithm for the uncapacitated facility location problem unless P = NP.
2. Give an $\mathcal{O}(\ln |D|)$-approximation algorithm for the uncapacitated facility location problem.

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The result of the struggle between the thought and
the ability to express it, between dream and reality,
is seldom more than a compromise or an approximation.
                                        - M.C. Escher
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