Technische Universität München
Fakultät für Informatik
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Summer term 2019
Assignment 4
May 13, 2017

Efficient Algorithms and Data Structures II

Deadline: May 20, 10:15 am in the ***Efficient Algorithms*** *folder.*

Homework 1 (5 Points)

Let (P) be a given feasible, bounded Linear Program. We know how to find the dual (D) of (P).

(a) By combining (P) and (D), demonstrate a linear program whose feasible solution correspond to feasible solutions optimizing the objective value of (P) (and similarly for (D)).

The new linear program should have constraints linear in the number of constraints of (P) and (D). Further, ensure that the new linear program is infeasible if either (P) or (D) is infeasible.

(b) Using the above idea demonstrate that we can reduce (in polynomial time,) the problem of solving an LP to that of finding whether an LP is feasible. In other words show that finding whether an LP is feasible is as tough (in terms of complexity) as solving an LP for optimality.

Homework 2 (5 Points)

Let A be a symmetric square matrix. Consider the linear programming problem

$$\begin{array}{lrcl} \text{min.} & c^T x & & \\ \text{s.t.} & Ax & \geq & c \\ & x & \geq & 0\ . \end{array}$$

Prove that if x^* satisfies $Ax^* = c$ and $x^* \geq 0$, then x^* is an optimal solution.

Homework 3 (5 Points)

Consider the LP from Homework 3, Sheet 3:

$$\begin{array}{lrcll} \text{max.} & \sum_e x_e & & & \\ \text{s.t.} & \sum_{e:\ e \text{ is incident to } v} x_e & \leq & 1 & \forall v \in V \\ & x_e & \geq & 0 & \forall e \in E \end{array}$$

(a) Write the dual (D) of the LP.

(b) Interpret the dual (D). What graph problem is modeled here?

(c) Show that (D) always has an optimum integral 0/1 solution.

Homework 4 (5 Points)

Consider the linear programming problem of minimizing $c^T x$ subject to $Ax = b, x \geq 0$. Let x^* be an optimal solution, assumed to exist, and let y^* be an optimal solution to the dual.

(a) Let $\bar{x}$ be an optimal solution to the primal, when the vector c is replaced by some other vector $\bar{c}$. Show that $(\bar{c} - c)^T(\bar{x} - x^*) \leq 0$.

(b) Let the cost vector be fixed at c, but suppose that we now change b to $\bar{b}$. Let $\bar{x}$ be the optimal primal solution corresponding to $\bar{b}$.

Prove that $(y^*)^T(\bar{b} - b) \leq c^T(\bar{x} - x^*)$.

> It's such a deep theme of the universe, duality
> - man woman, black white, high low, right wrong,
> up down, hello goodbye - that it was a very easy
> song to write. It's just a song of duality, with
> me advocating the more positive.
>
> \- P. McCartney