Technische Universität München
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Summer term 2019
Assignment 3
May 6, 2017

Efficient Algorithms and Data Structures II

*Deadline: May 13, 10:15 am in the **Efficient Algorithms** folder.*

Homework 1 (5 Points)

Use the simplex method to describe all the optimal solutions of the following problem

$$\begin{array}{lrcrcrcrcrcrcl} \text{max.} & 3x_1 & + & 2x_2 & + & 4x_3 & & & & & & & & \\ \text{s.t.} & x_1 & + & x_2 & + & 2x_3 & + & s_1 & & & & & = & 4 \\ & 2x_1 & & & + & 3x_3 & & & + & s_2 & & & = & 5 \\ & 2x_1 & + & x_2 & + & 3x_3 & & & & & + & s_3 & = & 7 \\ & & & & & \multicolumn{7}{c}{x_1, x_2, x_3, s_1, s_2, s_3} & \geq & 0\ . \end{array}$$

Homework 2 (5 Points)

Prove the following statements (where + denotes the Minkowski sum).

(a) X is an affine subspace if and only if $X = \emptyset$ or there is a linear subspace U and $v \in \mathbb{R}^n$ with $X = v + U$.

(b) Let P be a polyhedron. Any vertex of P is an extreme point of P.

Homework 3 (5 Points)

An $m \times n$ matrix is *totally unimodular* if the determinant of every square submatrix of A is $+1, -1$ or 0. It is known that if A is totally unimodular and b is integral, then the vertices of the polyhedron $P = \{x \mid Ax \leq b, x \geq 0\}$ are integral.
The following LP-relaxation models the maximum matching problem on a bipartite graph $G = (V_1 \cup V_2, E)$.

$$\begin{array}{llcll} \text{max.} & \sum_e x_e & & & \\ \text{s.t.} & \sum_{e\text{: } e \text{ is incident to } v} x_e & \leq & 1 & \forall v \in V \\ & x_e & \geq & 0 & \forall e \in E \end{array}$$

(a) Show that the above LP has a totally unimodular matrix.

(b) Show that the above LP always has an optimum solution that is 0/1-integral.

Homework 4 (5 Points)

Let $f, g_1, \ldots, g_k, h_1, \ldots, h_\ell : \mathbb{R}^n \to \mathbb{R}$ be continuously differentiable functions (that might not be linear). We consider the minimization problem

$$\begin{array}{llll} \text{min.} & f(x) & & \\ \text{s.t.} & g_i(x) & \leq & 0 \quad \forall i \in [k] \\ & h_j(x) & = & 0 \quad \forall j \in [\ell] \ . \end{array}$$

The *Lagrangian* $\mathcal{L}_x : \mathbb{R}^k \times \mathbb{R}^\ell \to \mathbb{R}$ at x is defined as

$$\mathcal{L}_x : (\lambda, \nu) \mapsto f(x) + \sum_{i=1}^{k} \lambda_i g_i(x) + \sum_{j=1}^{\ell} \nu_j h_j(x) \ .$$

The *Lagrangian Dual Problem* is the optimization problem

$$\max_{(\lambda,\nu) \in \mathbb{R}^k \times \mathbb{R}^\ell, \lambda \geq 0} \ \inf_{x \in \mathbb{R}^n} \mathcal{L}_x(\lambda, \nu) \ .$$

Consider the problem of minimizing the function $x^2 + y^2$ s.t. $x + y = 2$. Find the Lagrangian and the (sufficiently simplified) Lagrangian Dual Problem. Then solve the Lagrangian Dual Problem.

Simplex Sigillum Veri.
\- A. Schopenhauer