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Assignment 2
April 29, 2017

Efficient Algorithms and Data Structures II

*Deadline: May 6, 10:15 am in the **Efficient Algorithms** folder.*

Homework 1 (6 Points)

Consider the problem

$$\begin{aligned} \max. & \frac{c^T x + \alpha}{d^T x + \beta} \\ \text{s.t. } & Ax \le b \\ & x \ge 0 \ , \end{aligned}$$

for $c, d \in \mathbb{Q}^n, \alpha, \beta \in \mathbb{Q}, A \in \mathbb{Q}^{m \times n}, b \in \mathbb{Q}^m$.
Suppose that we know that the region $X = \{x \mid Ax \le b, x \ge 0\}$ is nonempty and bounded. Furthermore, you may assume that $d^T x + \beta > 0$ inside the feasible region X.
Give an algorithm using Linear Programming as a subroutine that allows to compute the optimal cost with arbitrary precision.

Homework 2 (5 Points)

Let F be the closed region whose boundary is obtained by connecting the following 'vertex' points in the $X - Y$ plane by a straight line segment:

$$(0,0), (0,6), (4,9), (3,5), (8,10), (9,6), (11,5), (7,0), (0,0)$$

The region F also includes the boundary points. We want to find a point in F which maximizes the linear objective function $x + y$.
Consider the following procedure:
Start from a vertex p (say $(0,0)$) and move to an adjacent vertex q such the value of our linear objective function is greater at q than at p. Call q our new current vertex.
Repeat this procedure with the current vertex till you can find an adjacent vertex which does not decrease the value of the linear objective function.
When this procedure terminates, are we left with a vertex maximizing the linear objective function? Why? What is the difference between this procedure and the simplex algorithm?

Homework 3 (2 Points)

In the simplex algorithm, we are at a current basis B. What does it mean that the coefficient of a non-basic variable in the objective function is 0?

Homework 4 (7 Points)

Consider the following LP:

$$\begin{array}{lllllllllll} \text{max.} & 10x_1 & - & 57x_2 & - & 9x_3 & - & 24x_4 & & & \\ \text{s.t.} & \frac{1}{2}x_1 & - & \frac{11}{2}x_2 & - & \frac{5}{2}x_3 & + & 9x_4 & + x_5 & & = 0 \\ & \frac{1}{2}x_1 & - & \frac{3}{2}x_2 & - & \frac{1}{2}x_3 & + & x_4 & & + x_6 & = 0 \\ & x_1 & & & & & & & & + x_7 & = 1 \\ & & & & & x_i & & & & & \geq 0 \qquad \text{for } i \in [7] \end{array}$$

Solve this LP by using the simplex method. If there are two or more choices for the row (variable leaving the basis) and column (nonbasic variable entering the basis), consider the following 2 approaches:

(a) Choose a column candidate with largest coefficient in the target function and choose the row candidate with the smallest index.

(b) Choose the column candidate with smallest index (amongst those having positive coefficients) in the target function and choose the row candidate with the smallest index. (This is also known as Bland's rule.)

Solve the LP using (a) as well as (b).

> I want to emphasize again that the greater part of the problems of which I shall speak [production planning using LPs], relating to the organization and planning of production, are connected specifically with the Soviet system of economy and in the majority of cases do not arise in the economy of a capitalist society.
>
> – L. V. Kantorovich