Technische Universität München
Fakultät für Informatik
Lehrstuhl für Algorithmen und Komplexität
Prof. Dr. Harald Räcke
Richard Stotz

Summer term 2019
Assignment 1
April 29, 2017

Efficient Algorithms and Data Structures II

Deadline: None, Tutorial exercises only.

Tutorial Exercise 1

A closed convex set is called line-free (geradenfrei) if it does not contain any (straight) line.

(a) Let P be the feasible region of an LP in standard form. Show that P is line free.

(b) Let P be the feasible region of an LP in the form $\max c^T x$ s.t. $Ax \leq b$ (natural form). Show that P might not be line free.

(c) Show that Theorem 21 does not hold if P is in natural form.

Tutorial Exercise 2

Consider the following LP:

$$\begin{array}{llrcrcrcr}
\text{max.} & & 2x_1 & - & 3x_2 & + & 3x_3 & & \\
\text{s.t.} & & x_1 & + & x_2 & - & x_3 & \leq & 7 \\
 & - & x_1 & - & x_2 & + & x_3 & \leq & -7 \\
 & & x_1 & - & 2x_2 & + & 2x_3 & \leq & 4 \\
 & & & & x_1, x_2, x_3 & & & \geq & 0\ .
\end{array}$$

(a) Rewrite the LP in standard form and then in matrix form.

(b) Give three feasible solutions and their objective value.

(Extra) Solve the LP using the Simplex algorithm shown in the lecture.

Tutorial Exercise 3

Formulate the minimum spanning tree problem on an edge-weighted graph $G = (V, E, c)$ as a integer linear program with $|E|$ variables, one for each edge.

Tutorial Exercise 4

Consider the data set found under `http://www14.in.tum.de/lehre/2019SS/ea/uebung/loesungen/data/ex1-4-data.csv` of students in some TUM course. Any student that passed the final exam has PASSED=1 (positive instance), any student that failed the exam has FAILED=-1 (negative instance). We want to determine a hyperplane that separates the students that failed from those that passed.
In order to do so, we want to construct a linear function h such that

$$h(p^i) \geq 1$$

for all positive instances p^i and

$$h(q^i) \leq -1$$

for all negative instances q^i. The *hinge loss* incurred by $h$is the extend to which the above inequalities fail to hold. For a positive instance p^i this is $\max\{1 - h(p^i), 0\}$, for a negative instance q^i this is $\max\{1 + h(q^i), 0\}$.
The linear function h is to be chosen so that the total hinge loss is minimal. In order to do so, we solve the following linear program:

$$\begin{array}{llcll} \text{min.} & \sum_i e_i & & & \\ \text{s.t.} & 1 - \left(\sum_{j=1}^d a_j p_j^i + b\right) & \leq & e_i & \text{for every negative instance } p^i \\ & 1 + \left(\sum_{j=1}^d a_j q_j^i + b\right) & \leq & e_i & \text{for every negative instance } q^i \\ & e_i & \geq & 0 & . \end{array}$$

Solve the given linear program using a method of your choice. It is maybe easiest to use Python Pulp `https://pythonhosted.org/PuLP/index.html` .

> The success of solving linear programming therefore depends on a number of factors: (1) the power of computers, (2) extremely clever algorithms; but it depends most of all upon (3) a lot of good luck that the matrices of practical problems will be very very sparse and that their bases, after rearrangement, will be nearly triangular.
>
> – G. B. Dantzig