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Assignment 14
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Efficient Algorithms and Data Structures I

Deadline: February 4, 2019, 10:15 am in the ***Efficient Algorithms*** *mailbox.*

Homework 1 (5 Points)

Let $G = (V = L \cup R, E)$ be an unweighted bipartite graph and let M be any matching in G. We say that node $v \in V$ is *covered* by M, if any edge in M is incident to node v.
We define the following integers:

- k_1 is the size of a largest matching M' that covers all vertices covered by M,
- k_2 is the size of a largest matching in G.

Clearly, $k_1 \leq k_2$. Show that $k_1 = k_2$.

Homework 2 (5 Points)

Suppose there exist n houses and n buyers for them. Buyer i values house j with $v(i,j)$ Euros. Each buyer wants to buy exactly one house.

(a) Describe how to find a distribution of the houses that maximizes the average perceived value for the buyers.

(b) The buyers are selfish and do not care about the average value. We therefore want to set prices $p(j)$ to the houses. Buyer i buys wants to buy any house that maximizes $v(i,j) - p(j)$. Describe a way to set the prices, so that each buyer wants to buy the house allocated to him in Step (a). Prove that your method works.

Hint: Think about the node weights in Weighted Bipartite Matching.

Homework 3 (5 Points)

As a violent thunderstorm is approaching, farmer Ferdinand wants to send his n harvesting drones $D_1, \ldots, D_n$ to take shelter in nearby hideouts. There exist k hideouts $h_1, \ldots, h_k$ in the region and each of the n drones can only reach a hideout that is less than 10 minutes away from its current position. Using hideout h_i costs c_i Euros per drone and hideout h_i has space for u_i drones. If drone D_j cannot reach a hideout in time, it must be repaired for d_j Euros.
Ferdinand knows all of the above information and the current positions of his drones. He needs to find an assignment of drones to hideouts that spends as little money as possible, even if that means that some drones do not reach a hideout and must be repaired.

(a) Model the above problem as a minimum cost flow problem. Describe precisely how your network is constructed and specify what a flow unit represents.

(b) Prove that a distribution of the drones to the hideouts implies an integral flow of the same cost in your network.

Homework 4 (5 Points)

During his daily stroll through the FMI building in Garching, the ghost Ambrosius finds a mysterious inscription on the whiteboard in some scientist's office. It is an $m \times n$ matrix, with $m > n$ filled with integers from $\{1,\ldots,m\}$. Ambrosius notes that no row or column contains the same number twice.
Ambrosius wants to prank the scientist by extending the matrix to an $m \times m$ matrix, effectively adding $m - n$ columns. To make his prank even funnier, he plans to fill the new columns with integers from $\{1,\ldots,m\}$ such that the new matrix still has no row or column containing the same number twice.
Show that Ambrosius' plan can succeed!
Hint: Use Hall's theorem!

Bonus Homework 1 (6 Bonus Points)

FAQ Time! Please send your questions to your tutor via email at least 48 hours before the tutorial.
You may earn bonus points for asking good questions.

Bonus Homework 2 (5 Bonus Points)

In another dimension, n goblin tribes are in trade with each other. For each tribe i, the value s_i is its budget balance: negative s_i indicates a deficit, while positive s_i indicates a surplus. Tribe i exports goods of value $e(i,j) \geq 0$ to tribe j.
When electing a new goblin king, the goblins follow a peculiar system. A subset S of tribes is *electable*, if the sum of the budget surpluses of the tribes in S, minus the total value of all exports from tribes in S to tribes not in S is nonnegative. If there exists an electable set of tribes, the new goblin king is chosen at random from the members of all electable tribes. If there is no electable set of tribes, no king is chosen.
Give a polynomial-time algorithm that decides if there is an electable set of tribes that is not equal to the full set. Prove that your method works!

Tutorial Exercise 1

A hospital needs to assign a group of n neurosurgeons to n patients. Each surgeon proposes, in decreasing order of preference, a list of three patients that he or she would like to perform surgery on. We want to determine whether there exists a *satisfiable assignment* (one that assigns the surgeons to the patients so that each surgeons obtains a patient on his or her list). If some satisfiable assignment is possible, we want to find the assignment that maximizes the number of surgeons with their most preferred patient, and further, among such assignments, the assignment that maximizes the number of patients with their second most preferred patient. Show how to solve this problem by solving a single assignment problem.

> If you find that you're spending almost all your time on theory, start turning some attention to practical things; it will improve your theories. If you find that you're spending almost all your time on practice, start turning some attention to theoretical things; it will improve your practice.
>
> – D. E. Knuth