Technische Universität München
Fakultät für Informatik
Lehrstuhl für Algorithmen und Komplexität
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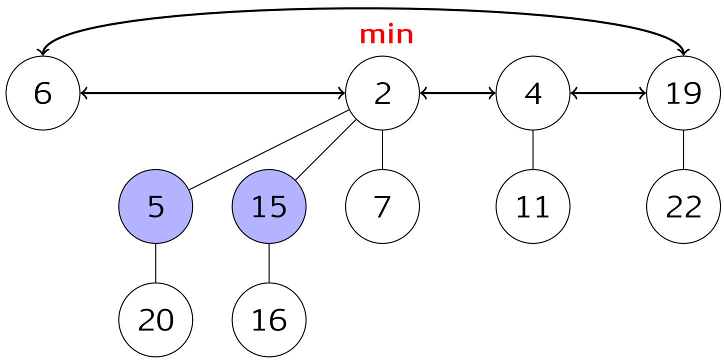
Winter term 2018/19
Assignment 11
January 7, 2019

Efficient Algorithms and Data Structures I

*Deadline: January 14, 2019, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (5 Points)

Perform the following operations sequentially on the Fibonacci Heap shown below so that it remains a Fibonacci Heap. Show what the heap looks like after each operation (always carry out each operation on the result of the previous operation).
Nodes that are marked appear in blue.



1. Insert(1)
2. DeleteMin()
3. Delete(19)
4. DecreaseKey(20,3)

Homework 2 (4 Points)

Consider the following sequence of $2n-1$ operations on a disjoint-set data structure with list implementation:

Makeset(x_1), Makeset(x_2),..., Makeset(x_n),
Union(x_2, x_1), Union(x_3, x_2), ..., Union(x_n, x_{n-1}).

1. Analyze the asymptotic running time of the sequence if the Union operation is implemented carelessly, appending the longer list to the shorter one.
2. Analyze the asymptotic running time of the sequence if the Union operation is implemented as shown in the lecture, appending the shorter list to the larger one.

Homework 3 (5 Points)

Let $G = (V, E)$ be a flow network with source s, sink t, and integer capacities. Let $n = |V|$ and $m = |E|$. Suppose that we are given a maximum flow in G.

(a) Suppose that the capacity of a single edge $(u, v) \in E$ is increased by 1. Give an $\mathcal{O}(m+n)$ time algorithm to update the maximum flow.

(b) Suppose that the capacity of a single edge $(u, v) \in E$ is decreased by 1. Give an $\mathcal{O}(m+n)$ time algorithm to update the maximum flow.

Homework 4 (6 Points)

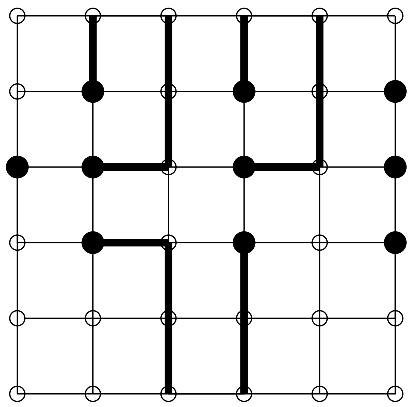
Archibald Anderson must coordinate the traffic of football fans from Munich central to the football stadium. He studies the effect of road blocks on the ability to transport the fans. He models the situation using a flow network. His manager asks him to determine the *most vital edge*, whose deletion causes the largest decrease in the maximum flow value.

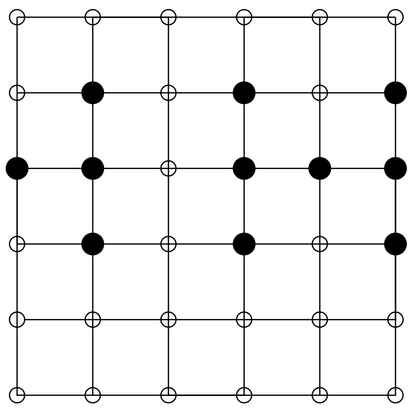
Let $f : E \to \mathbb{N}$ be a maximum flow in the network. Either prove or disprove (using a counterexample) the following claims

1. A most vital edge is an edge with maximum capacity.
2. A most vital edge is an edge with maximum value of $f(e)$.
3. A most vital edge is an edge whose flow $f(e)$ equals the maximum value of $f(e')$ among edges e' belonging to some minimum cut.
4. An edge that does not belong to some minimum cut cannot be a most vital edge.
5. A network might contain several most vital edges.

Tutorial Exercise 1

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices, as shown in the figures below. We denote the vertex in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1$, $i =$ n, $j = 1$, or $j =$ n. Given $m \leq n^2$ starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ in the grid, the escape problem is to determine whether or not there are m vertex-disjoint paths from the starting points to any m different points on the boundary. For example, the first grid below has an escape, but the second grid does not. Starting points are shown in black.





Describe an efficient algorithm to solve the escape problem, and analyze its running time.
Hint: You might need capacities at both nodes and edges. Argue that this is not a problem.

Tutorial Exercise 2

Consider a 0-1 matrix A with n rows and m columns. We refer to a row or a column of the matrix A as a *line*. We say that a set of 1's in the matrix A is *independent* if no two of them appear in the same line. We also say that a set of lines in the matrix is a *cover* of A if they cover all the 1's in the matrix. Using the max-flow min-cut theorem, show that the maximum number of independent 1's equals the minimum number of lines in a cover.

In any such war in Europe the rail network of Eastern Europe would be an important target. It therefore appears reasonable to illustrate the method [of using a flow network] by applying it to the Eastern European rail net.

- T.E. Harris, F.S. Ross