Technische Universität München
Fakultät für Informatik
Lehrstuhl für Algorithmen und Komplexität
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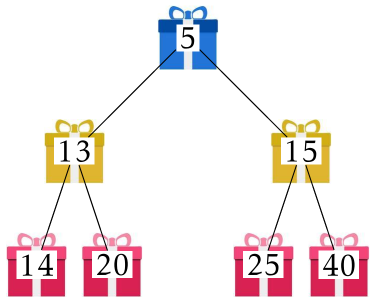
Winter term 2018/19
Assignment 10
December 17, 2018

Efficient Algorithms and Data Structures I

*Deadline: January 7, 2019, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (4 Points)

Cueball is preparing christmas by arranging his presents in a binary heap. All presents have an integer written on them. Initially, his heap looks like as follows:



(a) Some relatives arrive late with their presents and other relatives even retract their present. Cueball has to update his heap! Show how the heap looks like after each operation.

(i) Insert 10

(ii) Delete 40

(b) Cueball's sister Megan has had too much mulled wine. Now she asks herself how many valid heaps that hold exactly the set $\{5,13,14,20,15,25,40\}$ exist. Can you help?

Homework 2 (5 Points)

Santa's n elves $E_1,\ldots,E_n$ have a reindeer riding contest! At the start elves E_i and E_{i+1} are adjacent to each other. They start riding from a straight line at some angle ϕ_i (determined by their reindeer) and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever an elf E_j comes across the path traversed by any other elf E_i, we say that E_i defeated E_j and in that case, E_j stops riding.

(a) We call the point where E_i defeats E_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.

Assume here that all elves start in the same direction (all angles between 0 and 180 degrees), and that no more than 2 elves meet at the same point.

(b) Show how to enumerate in $\mathcal{O}(n\log n)$ time all events where one elf defeats another.

Homework 3 (5 Points)

You are attending the Christmas Party of the Fachschaft with n other students. You are – as usual – the first to arrive and the last one to leave. Student i arrives at time a_i and leaves at time ℓ_i.
Every student furthermore has a (distinct) christmas-factor c_i and your goal is to always talk with the most christmassy student in the room. If you are talking to someone and a student with higher christmas-factor comes to the party, you leave your current partner and talk to the newly arrived student. If your current partner leaves, you must find yourself a new partner.
You are dressed as a reindeer, so everyone wants to talk to you.

(a) Describe an efficient algorithm to decide at all times which person to talk to. You are aware of the values of c_i, a_i and ℓ_i of all people currently at the party, but you do not know who will arrive next.

(b) Sometimes the person you are talking to suddenly becomes ~~less~~ more christmassy, i.e., their c_i ~~decreases~~ increases. How can you adjust your data structure to this scenario in (amortized) constant time?

Homework 4 (6 Points)

Santa Claus says that $f(n) \in \overset{\infty}{\Omega}(g(n))$ if there exists a positive constant c such that

$$f(n) \geq c \cdot g(n) \geq 0 \qquad \text{for infinitely many integers } n.$$

(a) Give two nonnegative functions $f(n)$ and $g(n)$, such that $f(n) \in \overset{\infty}{\Omega}(g(n))$ but $f(n) \notin \Omega(g(n))$.

(b) Find inputs that cause DELETE-MIN, DECREASE-KEY, and DELETE to run in $\Omega(\log n)$ time for a binomial heap.

(c) Santa asks you to explain why running times of INSERT, MINIMUM, and MERGE are $\overset{\infty}{\Omega}(\log n)$ but not $\Omega(\log n)$ for a binomial heap. Will you help him?

Bonus Homework 1 (10 Bonus Points)

Note: Bonus points improve your score for both semester halves!
Answer the following questions. For true/false questions, you must explain your answer, otherwise no points are given.

(a) Suggest a quote for me to put at the end of the final exam. (Great answers get an extra award!)

(b) True/False: $2^n \in \Theta(2^{n+\log n})$.

(c) True/False: Inserting the keys 1, 2, 3, 4 and 5 into an initially empty (2,4)-tree always results in the same tree, no matter the order of insertion.

(d) True/False: There is no sequence of n inserts in an empty splay tree so that the resulting tree is a chain of length n.

(e) Which simple modification allows Binomial Heaps to have the MINIMUM() operation run in $\mathcal{O}(1)$?

Bonus Homework 2 (10 Bonus Points)

Coming Soon...

Tutorial Exercise 1

For any positive integer n, show a sequence of Fibonacci heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

Tutorial Exercise 2

Show that in a disjoint-set implementation using both union by rank and path compression, any sequence of m MAKESET, FIND and LINK operations takes only $O(m)$ time if all the LINK operations appear before any of the FIND operations.

> On December 25, Isaac Newton's birthday, we celebrate the existence of comprehensible physical laws. [...] One way to celebrate Grav-Mass is to decorate a tree with apples and other fruits. Glue them or attach them, but not too well! The idea is that occasionally a fruit should fall. Put them on the tree no more than 2 feet up, so that they won't get damaged or hurt anybody when they fall. Investigating and perfecting the methods for doing this is a great way expose a child to the process of scientifically studying the behavior of the physical world.
>
> \- R. Stallman