

18 Cuts & Metrics

Shortest Path

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall S \in \mathcal{S} \quad \sum_{e \in \delta(S)} x_e \geq 1 \\ & \forall e \in E \quad x_e \in \{0,1\} \end{array}$$

\mathcal{S} is the set of subsets that separate s from t .

The Dual:

$$\begin{array}{ll} \max & \sum_S y_S \\ \text{s.t.} & \forall e \in E \quad \sum_{S:e \in \delta(S)} y_S \leq c(e) \\ & \forall S \in \mathcal{S} \quad y_S \geq 0 \end{array}$$

The **Separation Problem** for the Shortest Path LP is the Minimum Cut Problem.

18 Cuts & Metrics

Shortest Path

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall S \in \mathcal{S} \quad \sum_{e \in \delta(S)} x_e \geq 1 \\ & \forall e \in E \quad x_e \geq 0 \end{array}$$

\mathcal{S} is the set of subsets that separate s from t .

The Dual:

$$\begin{array}{ll} \max & \sum_S y_S \\ \text{s.t.} & \forall e \in E \quad \sum_{S:e \in \delta(S)} y_S \leq c(e) \\ & \forall S \in \mathcal{S} \quad y_S \geq 0 \end{array}$$

The **Separation Problem** for the Shortest Path LP is the Minimum Cut Problem.

18 Cuts & Metrics

Shortest Path

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall S \in \mathcal{S} \quad \sum_{e \in \delta(S)} x_e \geq 1 \\ & \forall e \in E \quad x_e \geq 0 \end{array}$$

\mathcal{S} is the set of subsets that separate s from t .

The Dual:

$$\begin{array}{ll} \max & \sum_S y_S \\ \text{s.t.} & \forall e \in E \quad \sum_{S:e \in \delta(S)} y_S \leq c(e) \\ & \forall S \in \mathcal{S} \quad y_S \geq 0 \end{array}$$

The **Separation Problem** for the Shortest Path LP is the Minimum Cut Problem.

18 Cuts & Metrics

Minimum Cut

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall P \in \mathcal{P} \quad \sum_{e \in P} x_e \geq 1 \\ & \forall e \in E \quad x_e \in \{0,1\} \end{array}$$

\mathcal{P} is the set of path that connect s and t .

The Dual:

$$\begin{array}{ll} \max & \sum_P y_P \\ \text{s.t.} & \forall e \in E \quad \sum_{P:e \in P} y_P \leq c(e) \\ & \forall P \in \mathcal{P} \quad y_P \geq 0 \end{array}$$

The **Separation Problem** for the Minimum Cut LP is the Shortest Path Problem.

18 Cuts & Metrics

Minimum Cut

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall P \in \mathcal{P} \quad \sum_{e \in P} x_e \geq 1 \\ & \forall e \in E \quad x_e \geq 0 \end{array}$$

\mathcal{P} is the set of path that connect s and t .

The Dual:

$$\begin{array}{ll} \max & \sum_P y_P \\ \text{s.t.} & \forall e \in E \quad \sum_{P:e \in P} y_P \leq c(e) \\ & \forall P \in \mathcal{P} \quad y_P \geq 0 \end{array}$$

The Separation Problem for the Minimum Cut LP is the Shortest Path Problem.

18 Cuts & Metrics

Minimum Cut

$$\begin{array}{ll} \min & \sum_e c(e)x_e \\ \text{s.t.} & \forall P \in \mathcal{P} \quad \sum_{e \in P} x_e \geq 1 \\ & \forall e \in E \quad x_e \geq 0 \end{array}$$

\mathcal{P} is the set of path that connect s and t .

The Dual:

$$\begin{array}{ll} \max & \sum_P y_P \\ \text{s.t.} & \forall e \in E \quad \sum_{P:e \in P} y_P \leq c(e) \\ & \forall P \in \mathcal{P} \quad y_P \geq 0 \end{array}$$

The **Separation Problem** for the Minimum Cut LP is the Shortest Path Problem.

18 Cuts & Metrics

Minimum Cut

$$\begin{array}{ll} \min & \sum_e c(e)l_e \\ \text{s.t.} & \forall P \in \mathcal{P} \quad \sum_{e \in P} l_e \geq 1 \\ & \forall e \in E \quad l_e \geq 0 \end{array}$$

\mathcal{P} is the set of path that connect s and t .

The Dual:

$$\begin{array}{ll} \max & \sum_P f_P \\ \text{s.t.} & \forall e \in E \quad \sum_{P:e \in P} f_P \leq c(e) \\ & \forall P \in \mathcal{P} \quad f_P \geq 0 \end{array}$$

The **Separation Problem** for the Minimum Cut LP is the Shortest Path Problem.

18 Cuts & Metrics

Observations:

Suppose that ℓ_e -values are solution to Minimum Cut LP.

- ▶ We can view ℓ_e as defining the **length** of an edge.
- ▶ Define $d(u, v) = \min_{\text{path } P \text{ btw. } u \text{ and } v} \sum_{e \in P} \ell_e$ as the **Shortest Path Metric** induced by ℓ_e .
- ▶ We have $d(u, v) = \ell_e$ for every edge $e = (u, v)$, as otw. we could reduce ℓ_e without affecting the distance between s and t .

Remark for bean-counters:

d is not a metric on V but a semimetric as two nodes u and v could have distance zero.

18 Cuts & Metrics

Observations:

Suppose that ℓ_e -values are solution to Minimum Cut LP.

- ▶ We can view ℓ_e as defining the **length** of an edge.
- ▶ Define $d(u, v) = \min_{\text{path } P \text{ btw. } u \text{ and } v} \sum_{e \in P} \ell_e$ as the **Shortest Path Metric** induced by ℓ_e .
- ▶ We have $d(u, v) = \ell_e$ for every edge $e = (u, v)$, as otw. we could reduce ℓ_e without affecting the distance between s and t .

Remark for bean-counters:

d is not a metric on V but a semimetric as two nodes u and v could have distance zero.

18 Cuts & Metrics

Observations:

Suppose that ℓ_e -values are solution to Minimum Cut LP.

- ▶ We can view ℓ_e as defining the **length** of an edge.
- ▶ Define $d(u, v) = \min_{\text{path } P \text{ btw. } u \text{ and } v} \sum_{e \in P} \ell_e$ as the **Shortest Path Metric** induced by ℓ_e .
- ▶ We have $d(u, v) = \ell_e$ for every edge $e = (u, v)$, as otw. we could reduce ℓ_e without affecting the distance between s and t .

Remark for bean-counters:

d is not a metric on V but a semimetric as two nodes u and v could have distance zero.

18 Cuts & Metrics

Observations:

Suppose that ℓ_e -values are solution to Minimum Cut LP.

- ▶ We can view ℓ_e as defining the **length** of an edge.
- ▶ Define $d(u, v) = \min_{\text{path } P \text{ btw. } u \text{ and } v} \sum_{e \in P} \ell_e$ as the **Shortest Path Metric** induced by ℓ_e .
- ▶ We have $d(u, v) = \ell_e$ for every edge $e = (u, v)$, as otw. we could reduce ℓ_e without affecting the distance between s and t .

Remark for bean-counters:

d is not a metric on V but a semimetric as two nodes u and v could have distance zero.

How do we round the LP?

- ▶ Let $B(s, r)$ be the ball of radius r around s (w.r.t. metric d).
Formally:

$$B = \{v \in V \mid d(s, v) \leq r\}$$

- ▶ For $0 \leq r < 1$, $B(s, r)$ is an s - t -cut.

Which value of r should we choose? **choose randomly!!!**

Formally:

choose r **u.a.r.** (uniformly at random) from interval $[0, 1)$

How do we round the LP?

- ▶ Let $B(s, r)$ be the ball of radius r around s (w.r.t. metric d).
Formally:

$$B = \{v \in V \mid d(s, v) \leq r\}$$

- ▶ For $0 \leq r < 1$, $B(s, r)$ is an s - t -cut.

Which value of r should we choose? choose randomly!!!

Formally:

choose r u.a.r. (uniformly at random) from interval $[0, 1)$

How do we round the LP?

- ▶ Let $B(s, r)$ be the ball of radius r around s (w.r.t. metric d).
Formally:

$$B = \{v \in V \mid d(s, v) \leq r\}$$

- ▶ For $0 \leq r < 1$, $B(s, r)$ is an s - t -cut.

Which value of r should we choose? **choose randomly!!!**

Formally:

choose r u.a.r. (uniformly at random) from interval $[0, 1)$

How do we round the LP?

- ▶ Let $B(s, r)$ be the ball of radius r around s (w.r.t. metric d).
Formally:

$$B = \{v \in V \mid d(s, v) \leq r\}$$

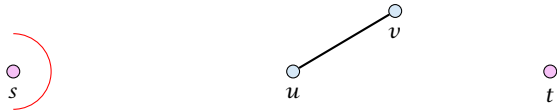
- ▶ For $0 \leq r < 1$, $B(s, r)$ is an s - t -cut.

Which value of r should we choose? **choose randomly!!!**

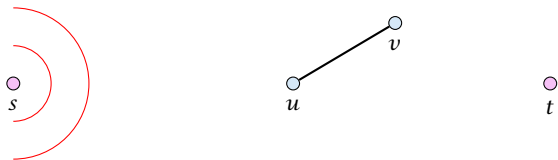
Formally:

choose r **u.a.r.** (**uniformly at random**) from interval $[0, 1)$

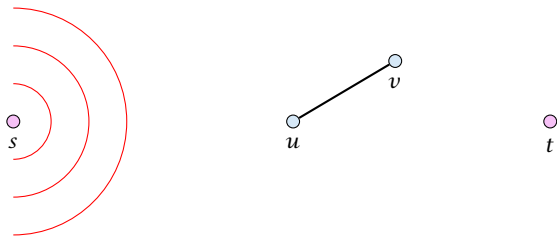
What is the probability that an edge (u, v) is in the cut?



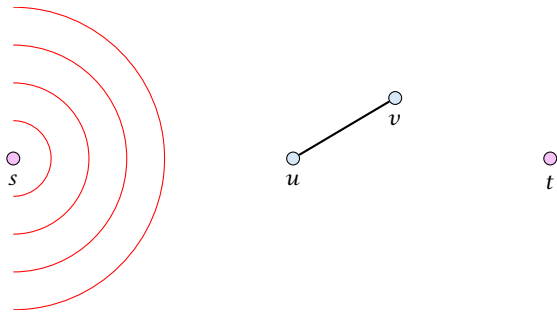
What is the probability that an edge (u, v) is in the cut?



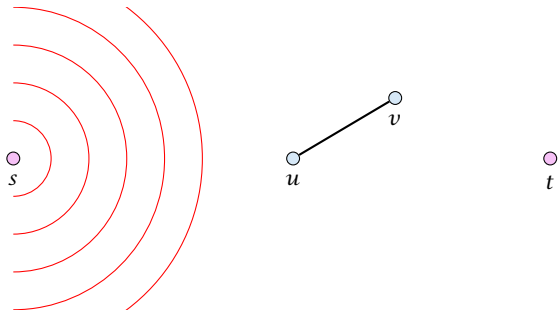
What is the probability that an edge (u, v) is in the cut?



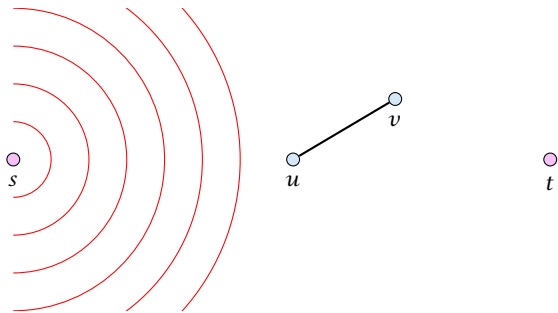
What is the probability that an edge (u, v) is in the cut?



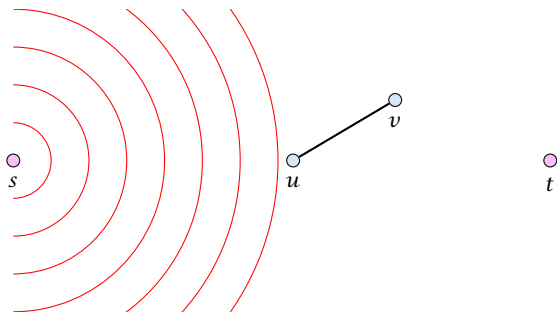
What is the probability that an edge (u, v) is in the cut?



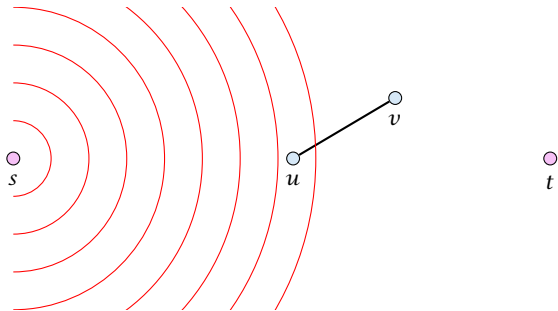
What is the probability that an edge (u, v) is in the cut?



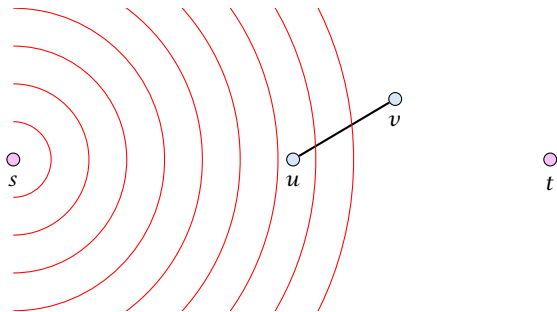
What is the probability that an edge (u, v) is in the cut?



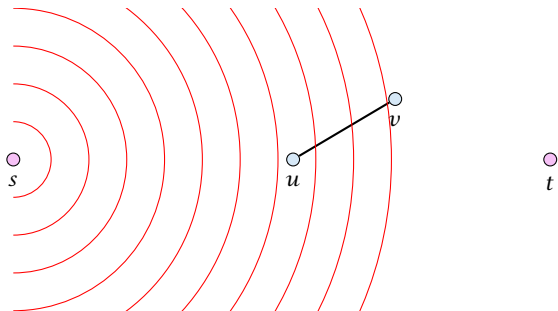
What is the probability that an edge (u, v) is in the cut?



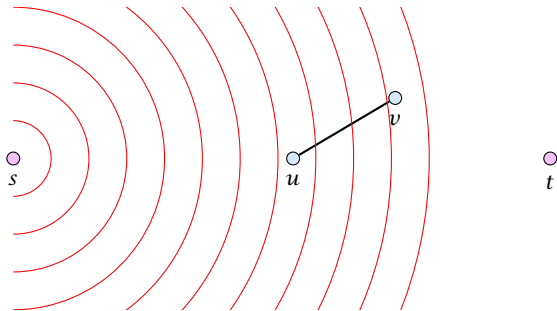
What is the probability that an edge (u, v) is in the cut?



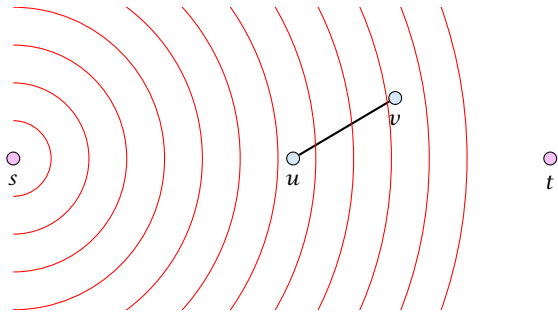
What is the probability that an edge (u, v) is in the cut?



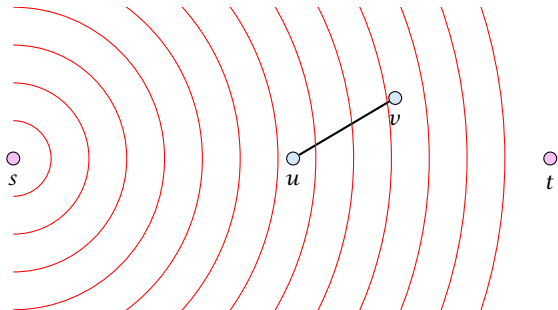
What is the probability that an edge (u, v) is in the cut?



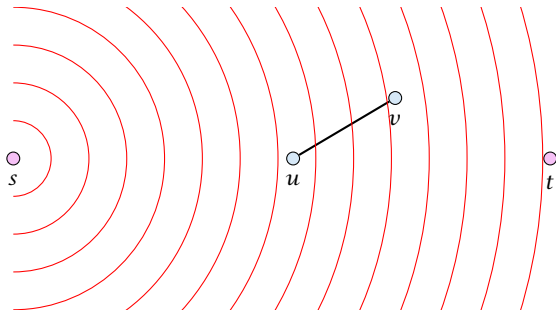
What is the probability that an edge (u, v) is in the cut?



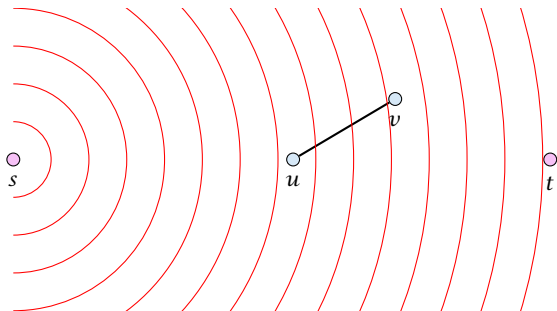
What is the probability that an edge (u, v) is in the cut?



What is the probability that an edge (u, v) is in the cut?



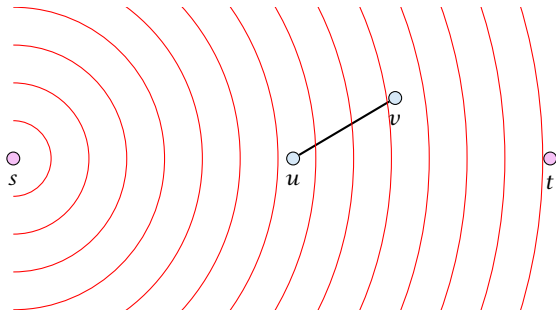
What is the probability that an edge (u, v) is in the cut?



▶ assume wlog. $d(s, u) \leq d(s, v)$

$\Pr[e \text{ is cut}]$

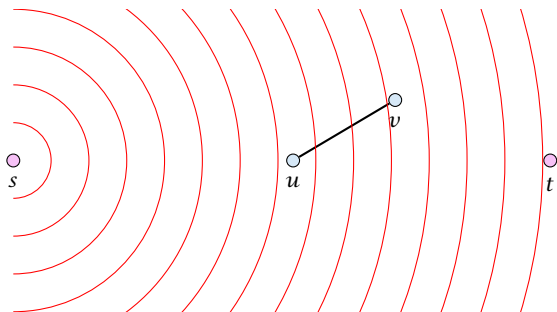
What is the probability that an edge (u, v) is in the cut?



▶ assume wlog. $d(s, u) \leq d(s, v)$

$$\Pr[e \text{ is cut}] = \Pr[r \in [d(s, u), d(s, v)]]$$

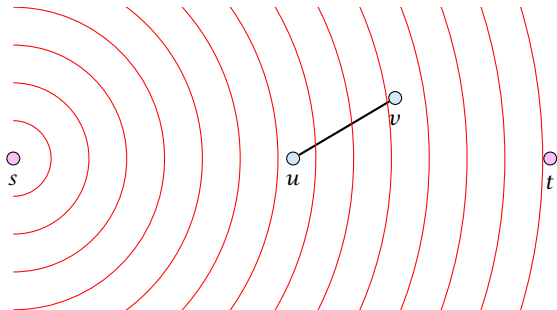
What is the probability that an edge (u, v) is in the cut?



► assume wlog. $d(s, u) \leq d(s, v)$

$$\Pr[e \text{ is cut}] = \Pr[r \in [d(s, u), d(s, v)]] \leq \frac{d(s, v) - d(s, u)}{1 - 0}$$

What is the probability that an edge (u, v) is in the cut?



► assume wlog. $d(s, u) \leq d(s, v)$

$$\begin{aligned} \Pr[e \text{ is cut}] &= \Pr[r \in [d(s, u), d(s, v)]] \leq \frac{d(s, v) - d(s, u)}{1 - 0} \\ &\leq \ell_e \end{aligned}$$

What is the expected size of a cut?

$$\begin{aligned} E[\text{size of cut}] &= E\left[\sum_e c(e) \Pr[e \text{ is cut}]\right] \\ &\leq \sum_e c(e) \ell_e \end{aligned}$$

On the other hand:

$$\sum_e c(e) \ell_e \leq \text{size of mincut}$$

as the ℓ_e are the solution to the Mincut LP *relaxation*.

Hence, our rounding gives an optimal solution.

What is the expected size of a cut?

$$\begin{aligned} E[\text{size of cut}] &= E\left[\sum_e c(e) \Pr[e \text{ is cut}]\right] \\ &\leq \sum_e c(e) \ell_e \end{aligned}$$

On the other hand:

$$\sum_e c(e) \ell_e \leq \text{size of mincut}$$

as the ℓ_e are the solution to the Mincut LP *relaxation*.

Hence, our rounding gives an optimal solution.

What is the expected size of a cut?

$$\begin{aligned} E[\text{size of cut}] &= E\left[\sum_e c(e) \Pr[e \text{ is cut}]\right] \\ &\leq \sum_e c(e) \ell_e \end{aligned}$$

On the other hand:

$$\sum_e c(e) \ell_e \leq \text{size of mincut}$$

as the ℓ_e are the solution to the Mincut LP *relaxation*.

Hence, our rounding gives an optimal solution.

Minimum Multicut:

Given a graph $G = (V, E)$, together with source-target pairs s_i, t_i , $i = 1, \dots, k$, and a capacity function $c : E \rightarrow \mathbb{R}^+$ on the edges. Find a subset $F \subseteq E$ of the edges such that all s_i-t_i pairs lie in different components in $G = (V, E \setminus F)$.

$$\begin{array}{ll} \min & \sum_e c(e) l_e \\ \text{s.t.} & \forall P \in \mathcal{P}_i \text{ for some } i \quad \sum_{e \in P} l_e \geq 1 \\ & \forall e \in E \quad l_e \in [0, 1] \end{array}$$

Here \mathcal{P}_i contains all path P between s_i and t_i .

Minimum Multicut:

Given a graph $G = (V, E)$, together with source-target pairs s_i, t_i , $i = 1, \dots, k$, and a capacity function $c : E \rightarrow \mathbb{R}^+$ on the edges. Find a subset $F \subseteq E$ of the edges such that all s_i - t_i pairs lie in different components in $G = (V, E \setminus F)$.

$$\begin{array}{ll} \min & \sum_e c(e) \ell_e \\ \text{s.t.} & \forall P \in \mathcal{P}_i \text{ for some } i \quad \sum_{e \in P} \ell_e \geq 1 \\ & \forall e \in E \quad \ell_e \in \{0, 1\} \end{array}$$

Here \mathcal{P}_i contains all path P between s_i and t_i .

Minimum Multicut:

Given a graph $G = (V, E)$, together with source-target pairs s_i, t_i , $i = 1, \dots, k$, and a capacity function $c : E \rightarrow \mathbb{R}^+$ on the edges. Find a subset $F \subseteq E$ of the edges such that all s_i - t_i pairs lie in different components in $G = (V, E \setminus F)$.

$$\begin{array}{ll} \min & \sum_e c(e) \ell_e \\ \text{s.t.} & \forall P \in \mathcal{P}_i \text{ for some } i \quad \sum_{e \in P} \ell_e \geq 1 \\ & \forall e \in E \quad \ell_e \in \{0, 1\} \end{array}$$

Here \mathcal{P}_i contains all path P between s_i and t_i .

Re-using the analysis for the single-commodity case is difficult.

$$\Pr[e \text{ is cut}] \leq ?$$

- ▶ If for some R the balls $B(s_i, R)$ are disjoint between different sources, we get a $1/R$ approximation.
- ▶ However, this cannot be guaranteed.

Re-using the analysis for the single-commodity case is difficult.

$$\Pr[e \text{ is cut}] \leq ?$$

- ▶ If for some R the balls $B(s_i, R)$ are disjoint between different sources, we get a $1/R$ approximation.
- ▶ However, this cannot be guaranteed.

Re-using the analysis for the single-commodity case is difficult.

$$\Pr[e \text{ is cut}] \leq ?$$

- ▶ If for some R the balls $B(s_i, R)$ are disjoint between different sources, we get a $1/R$ approximation.
- ▶ However, this cannot be guaranteed.

- ▶ Assume for simplicity that all edge-length ℓ_e are multiples of $\delta \ll 1$.
- ▶ Replace the graph G by a graph G' , where an edge of length ℓ_e is replaced by ℓ_e/δ edges of length δ .
- ▶ Let $B(s_i, z)$ be the ball in G' that contains nodes v with distance $d(s_i, v) \leq z\delta$.

Algorithm 1 RegionGrowing(s_i, p)

```
1:  $z \leftarrow 0$   
2: repeat  
3:   flip a coin ( $\text{Pr}[\text{heads}] = p$ )  
4:    $z \leftarrow z + 1$   
5: until heads  
6: return  $B(s_i, z)$ 
```

- ▶ Assume for simplicity that all edge-length ℓ_e are multiples of $\delta \ll 1$.
- ▶ Replace the graph G by a graph G' , where an edge of length ℓ_e is replaced by ℓ_e/δ edges of length δ .
- ▶ Let $B(s_i, z)$ be the ball in G' that contains nodes v with distance $d(s_i, v) \leq z\delta$.

Algorithm 1 RegionGrowing(s_i, p)

```
1:  $z \leftarrow 0$ 
2: repeat
3:   flip a coin ( $\text{Pr}[\text{heads}] = p$ )
4:    $z \leftarrow z + 1$ 
5: until heads
6: return  $B(s_i, z)$ 
```

- ▶ Assume for simplicity that all edge-length ℓ_e are multiples of $\delta \ll 1$.
- ▶ Replace the graph G by a graph G' , where an edge of length ℓ_e is replaced by ℓ_e/δ edges of length δ .
- ▶ Let $B(s_i, z)$ be the ball in G' that contains nodes v with distance $d(s_i, v) \leq z\delta$.

Algorithm 1 RegionGrowing(s_i, p)

```
1:  $z \leftarrow 0$ 
2: repeat
3:     flip a coin ( $\text{Pr}[\text{heads}] = p$ )
4:      $z \leftarrow z + 1$ 
5: until heads
6: return  $B(s_i, z)$ 
```

- ▶ Assume for simplicity that all edge-length ℓ_e are multiples of $\delta \ll 1$.
- ▶ Replace the graph G by a graph G' , where an edge of length ℓ_e is replaced by ℓ_e/δ edges of length δ .
- ▶ Let $B(s_i, z)$ be the ball in G' that contains nodes v with distance $d(s_i, v) \leq z\delta$.

Algorithm 1 RegionGrowing(s_i, p)

```
1:  $z \leftarrow 0$ 
2: repeat
3:   flip a coin ( $\text{Pr}[\text{heads}] = p$ )
4:    $z \leftarrow z + 1$ 
5: until heads
6: return  $B(s_i, z)$ 
```

Algorithm 1 Multicut(G')

```
1: while  $\exists s_i-t_i$  pair in  $G'$  do  
2:    $C \leftarrow \text{RegionGrowing}(s_i, p)$   
3:    $G' = G' \setminus C$  // cuts edges leaving  $C$   
4: return  $B(s_i, z)$ 
```

- ▶ probability of cutting an edge is only p
- ▶ a source either does not reach an edge during Region Growing; then it is not cut
- ▶ if it reaches the edge then it either **cuts** the edge or **protects** the edge from being cut by other sources
- ▶ if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most OPT.

Algorithm 1 Multicut(G')

```
1: while  $\exists s_i-t_i$  pair in  $G'$  do  
2:    $C \leftarrow \text{RegionGrowing}(s_i, p)$   
3:    $G' = G' \setminus C$  // cuts edges leaving  $C$   
4: return  $B(s_i, z)$ 
```

- ▶ probability of cutting an edge is only p
- ▶ a source either does not reach an edge during Region Growing; then it is not cut
- ▶ if it reaches the edge then it either **cuts** the edge or **protects** the edge from being cut by other sources
- ▶ if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most OPT.

Algorithm 1 Multicut(G')

```
1: while  $\exists s_i-t_i$  pair in  $G'$  do  
2:    $C \leftarrow \text{RegionGrowing}(s_i, p)$   
3:    $G' = G' \setminus C$  // cuts edges leaving  $C$   
4: return  $B(s_i, z)$ 
```

- ▶ probability of cutting an edge is only p
- ▶ a source either does not reach an edge during Region Growing; then it is not cut
 - ▶ if it reaches the edge then it either **cuts** the edge or **protects** the edge from being cut by other sources
 - ▶ if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most OPT.

Algorithm 1 Multicut(G')

```
1: while  $\exists s_i-t_i$  pair in  $G'$  do  
2:    $C \leftarrow \text{RegionGrowing}(s_i, p)$   
3:    $G' = G' \setminus C$  // cuts edges leaving  $C$   
4: return  $B(s_i, z)$ 
```

- ▶ probability of cutting an edge is only p
- ▶ a source either does not reach an edge during Region Growing; then it is not cut
- ▶ if it reaches the edge then it either **cuts** the edge or **protects** the edge from being cut by other sources
- ▶ if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most OPT.

Algorithm 1 Multicut(G')

```
1: while  $\exists s_i-t_i$  pair in  $G'$  do  
2:    $C \leftarrow \text{RegionGrowing}(s_i, p)$   
3:    $G' = G' \setminus C$  // cuts edges leaving C  
4: return  $B(s_i, z)$ 
```

- ▶ probability of cutting an edge is only p
- ▶ a source either does not reach an edge during Region Growing; then it is not cut
- ▶ if it reaches the edge then it either **cuts** the edge or **protects** the edge from being cut by other sources
- ▶ if we choose $p = \delta$ the probability of cutting an edge is only its LP-value; our expected cost are at most **OPT**.

Problem:

We may not cut all source-target pairs.

A component that we remove may contain an s_i-t_i pair.

If we ensure that we cut before reaching radius $1/2$ we are in good shape.

Problem:

We may not cut all source-target pairs.

A component that we remove may contain an s_i-t_i pair.

If we ensure that we cut before reaching radius $1/2$ we are in good shape.

Problem:

We may not cut all source-target pairs.

A component that we remove may contain an s_i-t_i pair.

If we ensure that we cut before reaching radius $1/2$ we are in good shape.

- ▶ choose $p = 6 \ln k \cdot \delta$
- ▶ we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- ▶ we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

$$\Pr[\text{not successful}] \leq (1-p)^{\frac{1}{2\delta}} = \left((1-p)^{1/p} \right)^{\frac{p}{2\delta}} \leq e^{-\frac{p}{2\delta}} \leq \frac{1}{k^3}$$

- ▶ Hence,

$$\Pr[\exists i \text{ that is not successful}] \leq \frac{1}{k^2}$$

- ▶ choose $p = 6 \ln k \cdot \delta$
- ▶ we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- ▶ we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

$$\Pr[\text{not successful}] \leq (1-p)^{\frac{1}{2\delta}} = \left((1-p)^{1/p} \right)^{\frac{p}{2\delta}} \leq e^{-\frac{p}{2\delta}} \leq \frac{1}{k^3}$$

- ▶ Hence,

$$\Pr[\exists i \text{ that is not successful}] \leq \frac{1}{k^2}$$

- ▶ choose $p = 6 \ln k \cdot \delta$
- ▶ we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- ▶ we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

$$\Pr[\text{not successful}] \leq (1-p)^{\frac{1}{2\delta}} = \left((1-p)^{1/p} \right)^{\frac{p}{2\delta}} \leq e^{-\frac{p}{2\delta}} \leq \frac{1}{k^3}$$

- ▶ Hence,

$$\Pr[\exists i \text{ that is not successful}] \leq \frac{1}{k^2}$$

- ▶ choose $p = 6 \ln k \cdot \delta$
- ▶ we make $\frac{1}{2\delta}$ trials before reaching radius $1/2$.
- ▶ we say a Region Growing is not successful if it does not terminate before reaching radius $1/2$.

$$\Pr[\text{not successful}] \leq (1-p)^{\frac{1}{2\delta}} = \left((1-p)^{1/p} \right)^{\frac{p}{2\delta}} \leq e^{-\frac{p}{2\delta}} \leq \frac{1}{k^3}$$

- ▶ Hence,

$$\Pr[\exists i \text{ that is not successful}] \leq \frac{1}{k^2}$$

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

$$\begin{aligned} E[\text{cutsize} \mid \text{succ.}] &= \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ &\leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT} \end{aligned}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

$$\begin{aligned} E[\text{cutsize} \mid \text{succ.}] &= \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ &\leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT} \end{aligned}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

$$\begin{aligned} E[\text{cutsize} \mid \text{succ.}] &= \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ &\leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT} \end{aligned}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$E[\text{cutsize}] = \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}]$$

$$E[\text{cutsize} \mid \text{succ.}] = \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ \leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

$$\begin{aligned} E[\text{cutsize} \mid \text{succ.}] &= \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ &\leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT} \end{aligned}$$

Note: success means all source-target pairs separated

We assume $k \geq 2$.

What is expected cost?

$$\begin{aligned} E[\text{cutsize}] &= \Pr[\text{success}] \cdot E[\text{cutsize} \mid \text{success}] \\ &\quad + \Pr[\text{no success}] \cdot E[\text{cutsize} \mid \text{no success}] \end{aligned}$$

$$\begin{aligned} E[\text{cutsize} \mid \text{succ.}] &= \frac{E[\text{cutsize}] - \Pr[\text{no succ.}] \cdot E[\text{cutsize} \mid \text{no succ.}]}{\Pr[\text{success}]} \\ &\leq \frac{E[\text{cutsize}]}{\Pr[\text{success}]} \leq \frac{1}{1 - \frac{1}{k^2}} 6 \ln k \cdot \text{OPT} \leq 8 \ln k \cdot \text{OPT} \end{aligned}$$

Note: **success** means all source-target pairs separated

We assume $k \geq 2$.

If we are not successful we simply perform a trivial k -approximation.

This only increases the expected cost by at most $\frac{1}{k^2} \cdot kOPT \leq OPT/k$.

Hence, our final cost is $\mathcal{O}(\ln k) \cdot OPT$ in expectation.