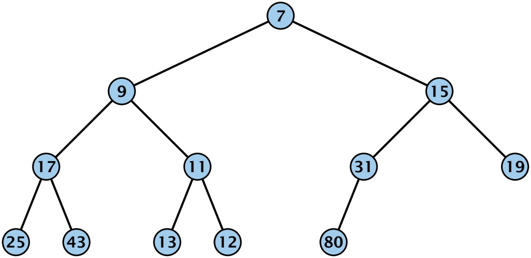
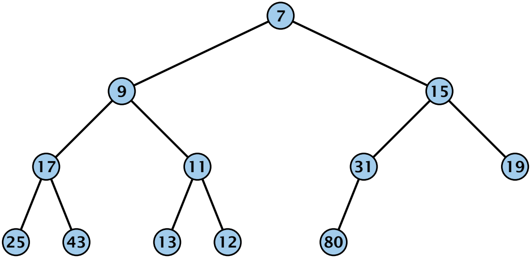
8.1 Binary Heaps



8.1 Binary Heaps

- Nearly complete binary tree; only the last level is not full, and this one is filled from left to right.



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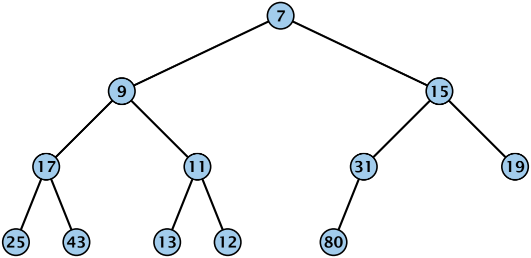
- Nearly complete binary tree; only the last level is not full, and this one is filled from left to right.
- Heap property: A node's key is not larger than the key of one of its children.



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Operations:

- **minimum()**: return the root-element. Time $\mathcal{O}(1)$.
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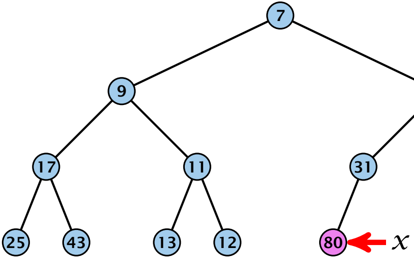
Maintain a pointer to the last element x.

- We can compute the predecessor of x (last element when x is deleted) in time $\mathcal{O}(\log n)$.

  go up until the last edge used was a right edge.
  go left; go right until you reach a leaf

  if you hit the root on the way up, go to the rightmost element



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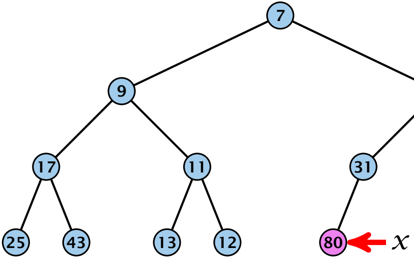

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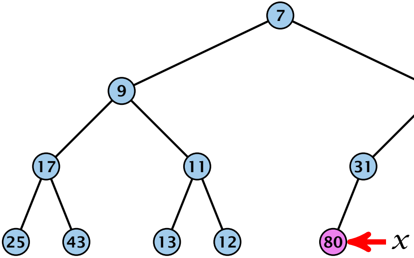

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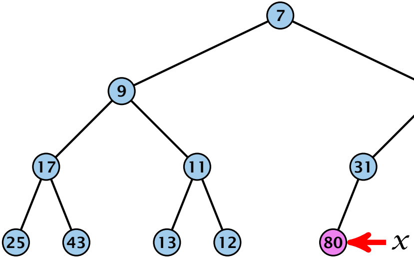

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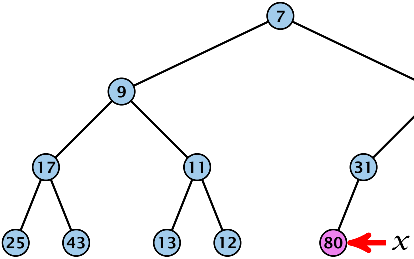

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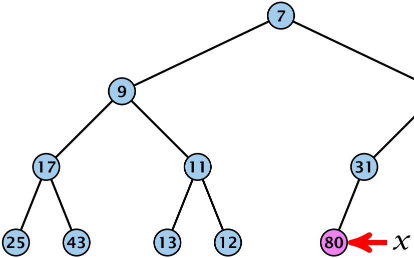

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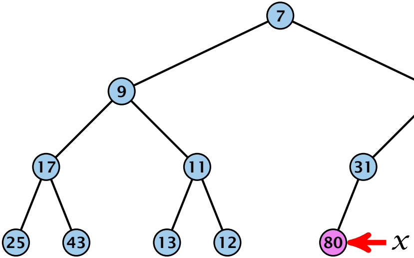

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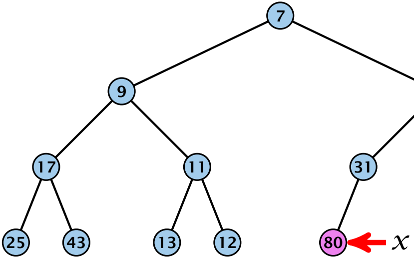

Insert

1. Insert element at successor of x.
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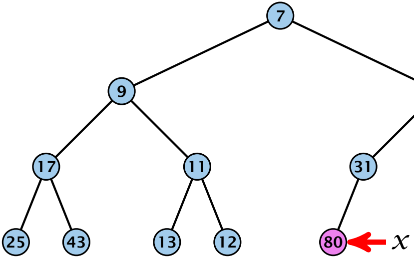

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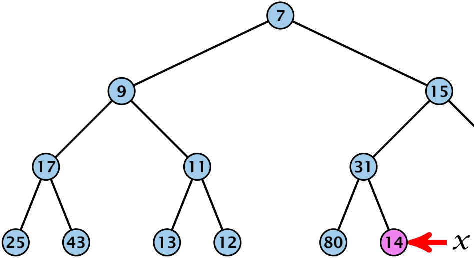

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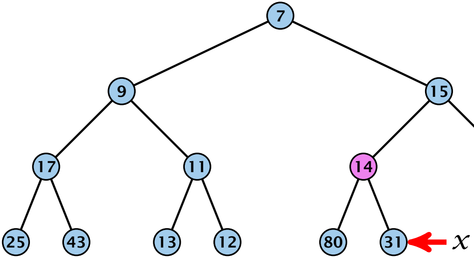

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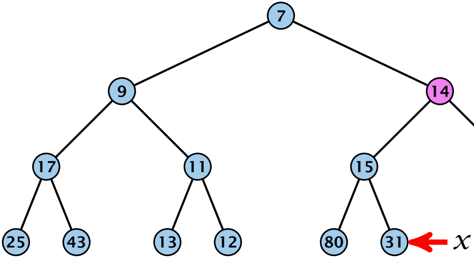

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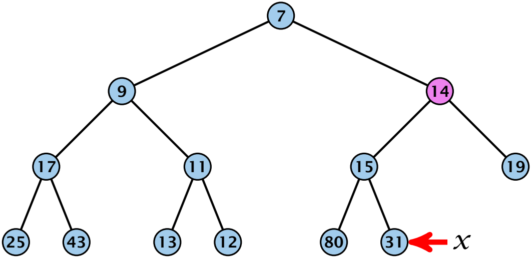

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Delete

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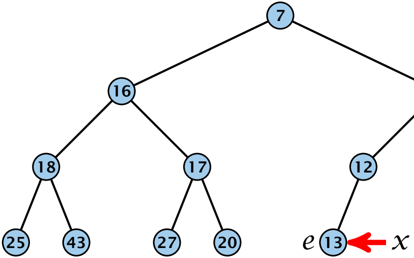

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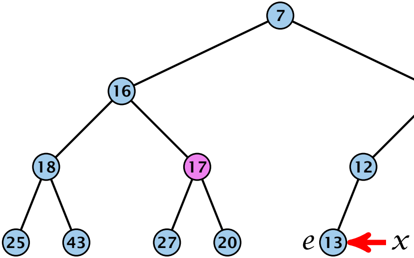

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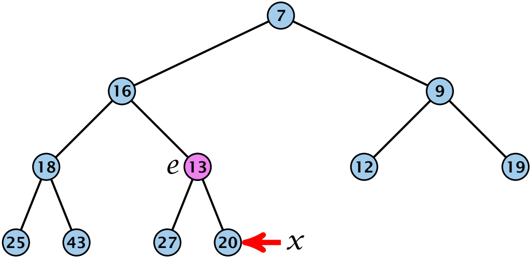

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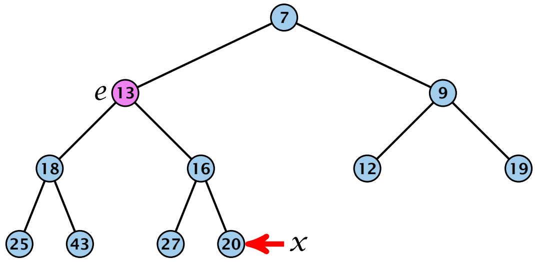

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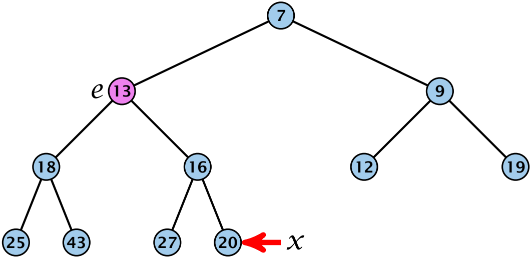

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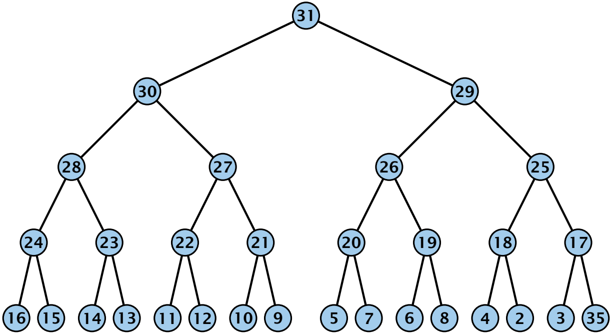
Binary Heaps

Operations:

- $\textbf{minimum()}$: return the root-element. Time $\mathcal{O}(1)$.
- $\textbf{is-empty()}$: check whether root-pointer is null. Time $\mathcal{O}(1)$.
- $\textbf{insert}(\boldsymbol{k})$: insert at successor of x and bubble up. Time $\mathcal{O}(\log n)$.
- $\textbf{delete}(\boldsymbol{h})$: swap with x and bubble up or sift-down. Time $\mathcal{O}(\log n)$.

Build Heap

We can build a heap in linear time:



$$\sum_{\text{levels } \ell} 2^{\ell} \cdot (h - \ell) = \sum_{i} i 2^{h-i} = \mathcal{O}(2^h) = \mathcal{O}(n)$$

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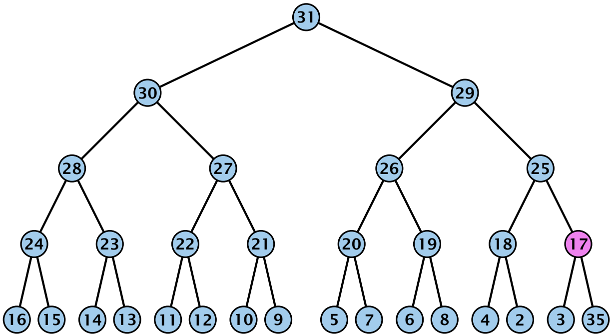

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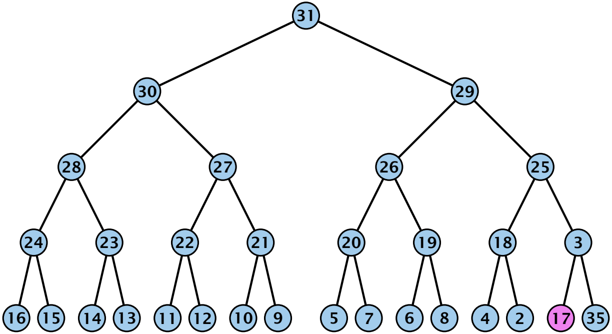

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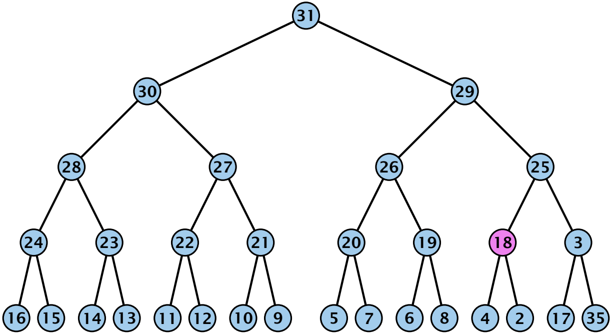

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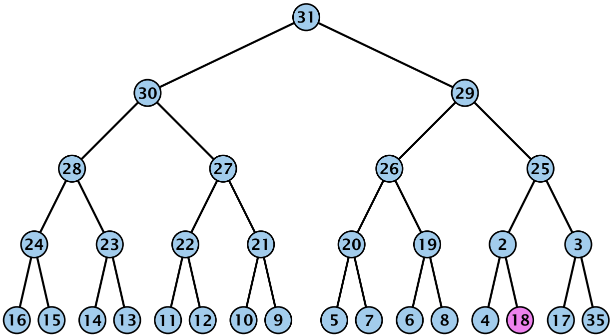

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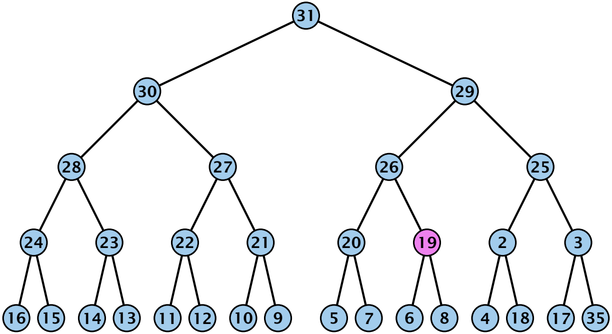

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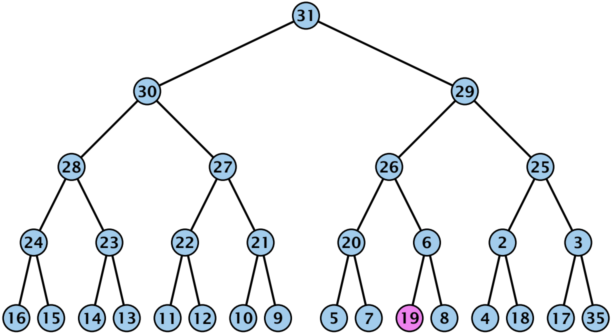

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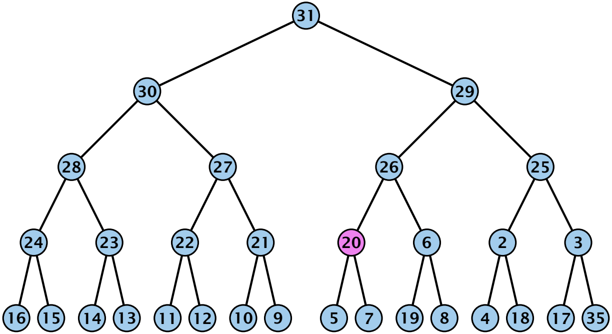

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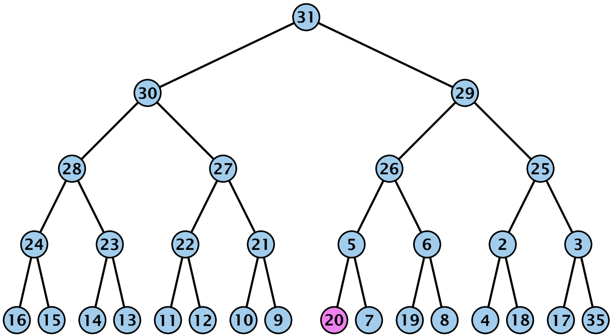

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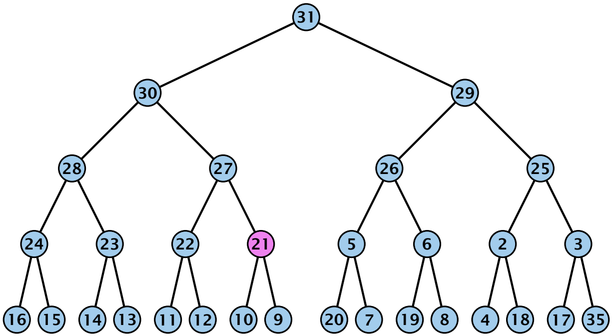

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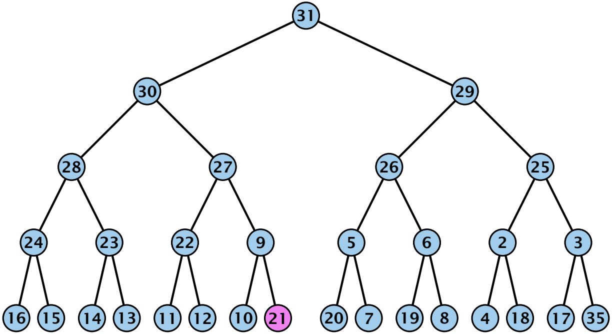

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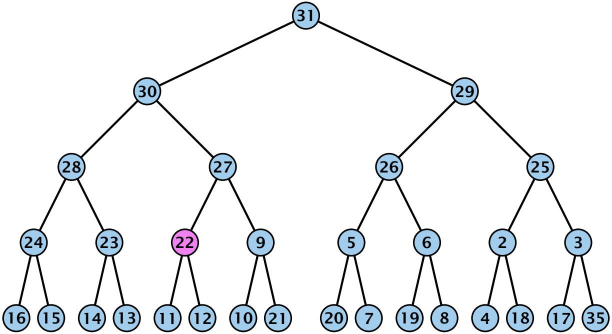

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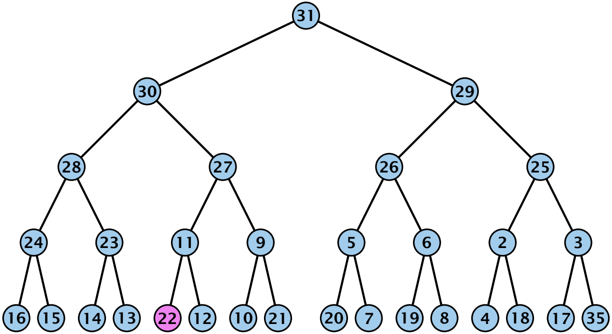

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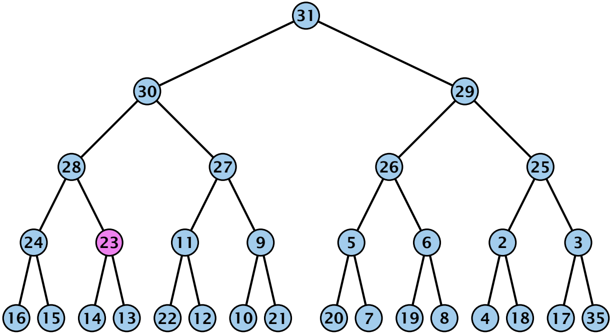

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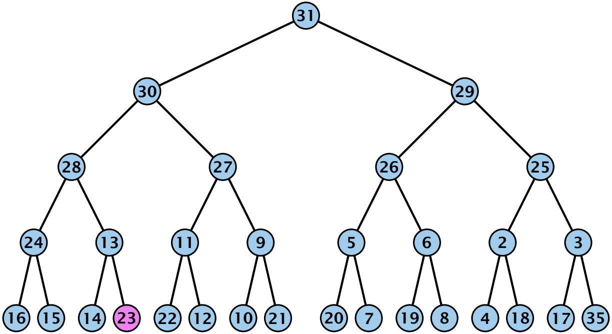

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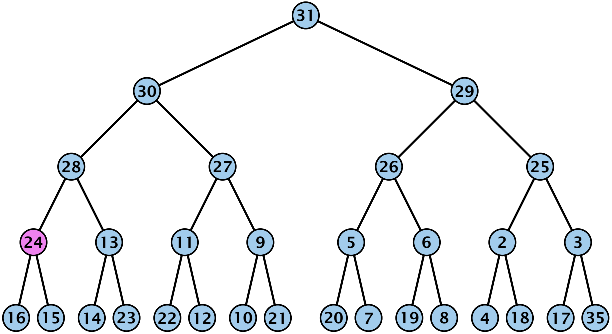

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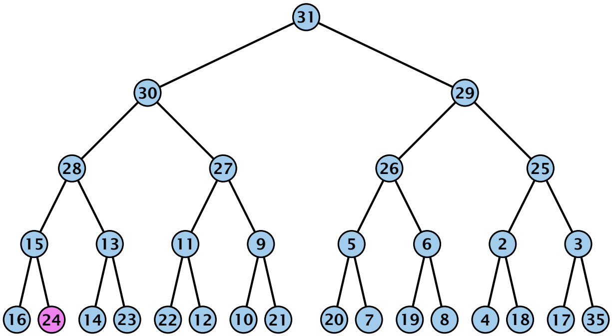

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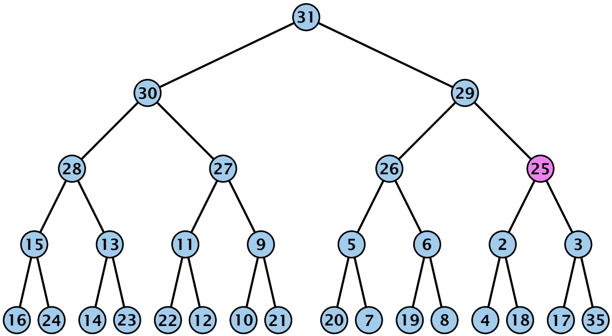

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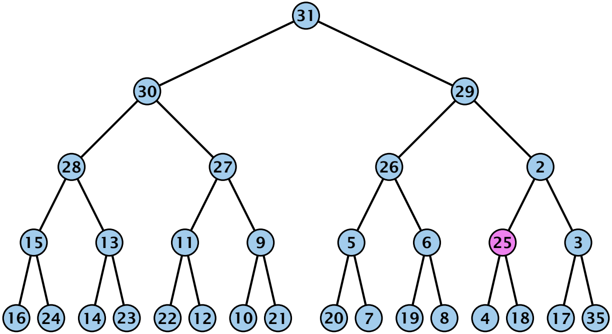

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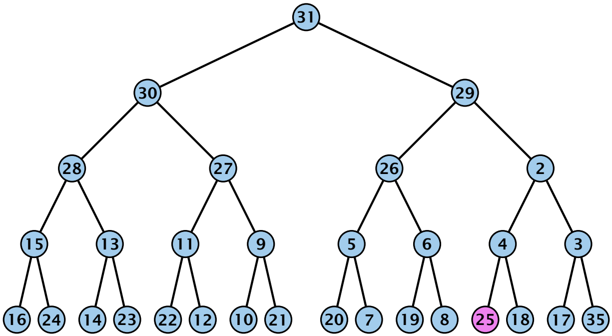

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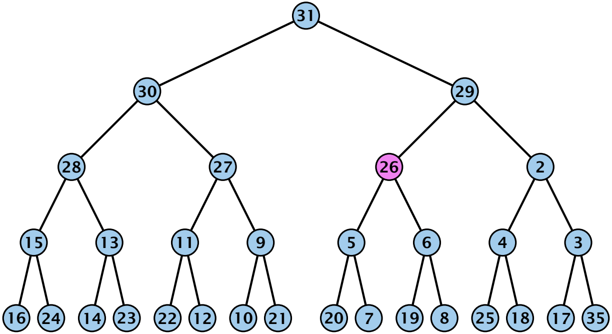

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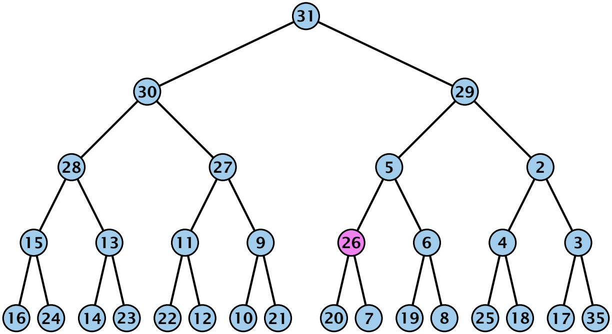

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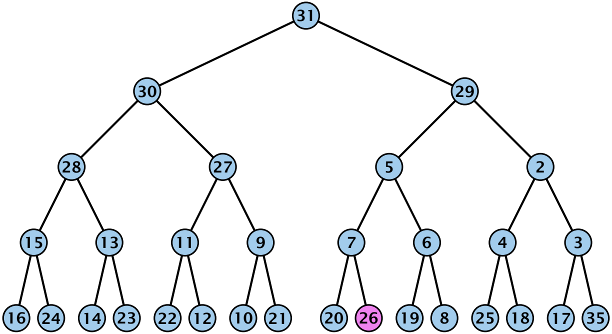

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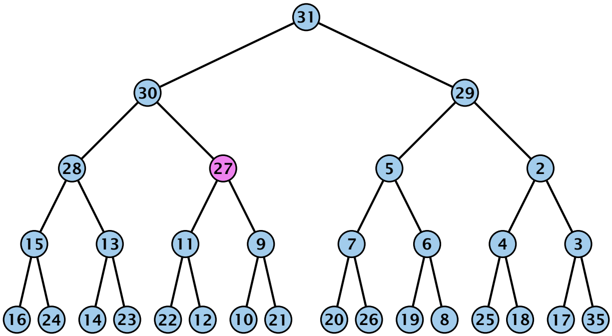

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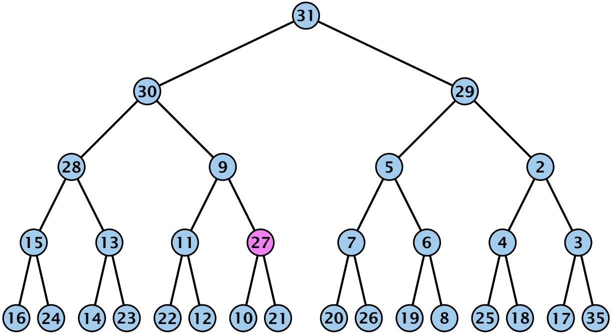

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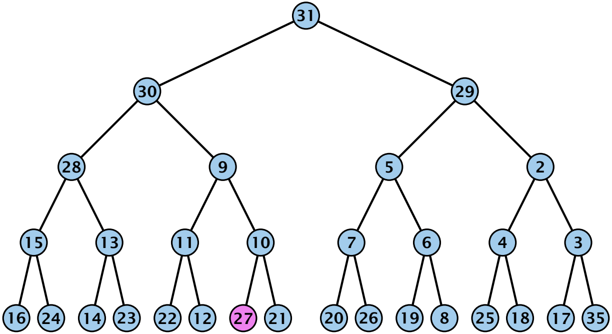

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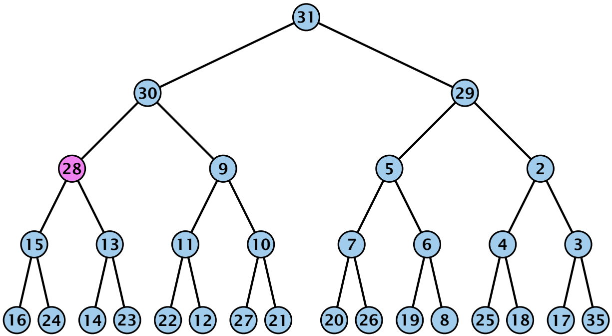

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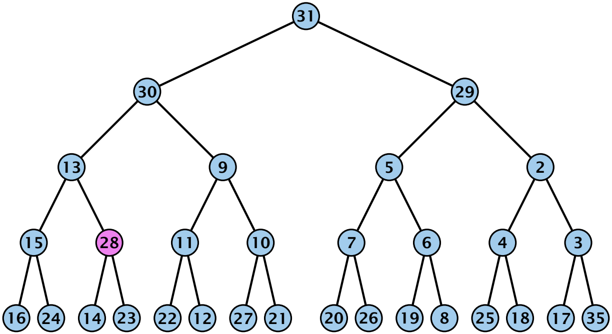

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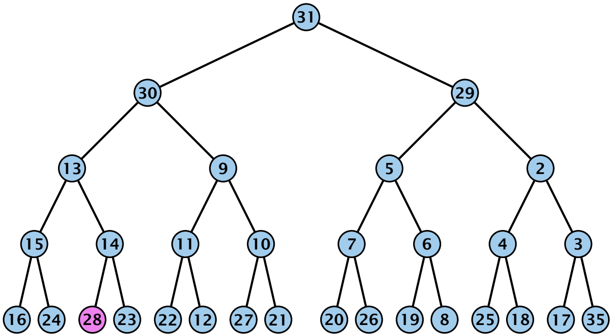

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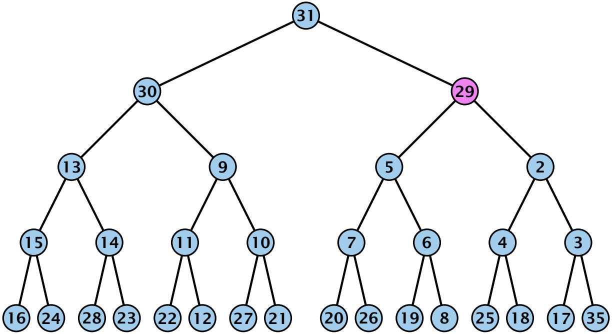

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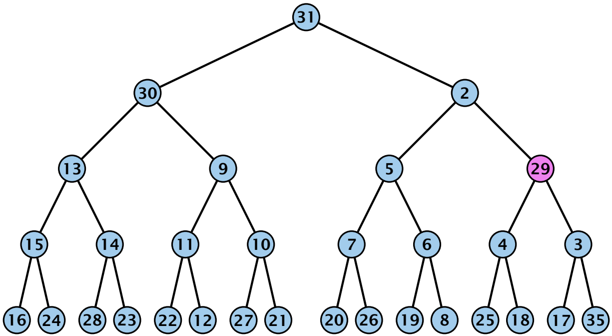

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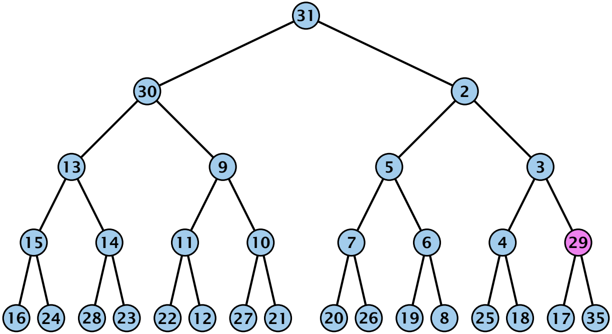

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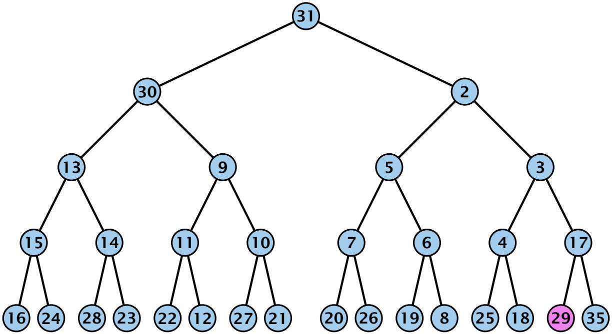

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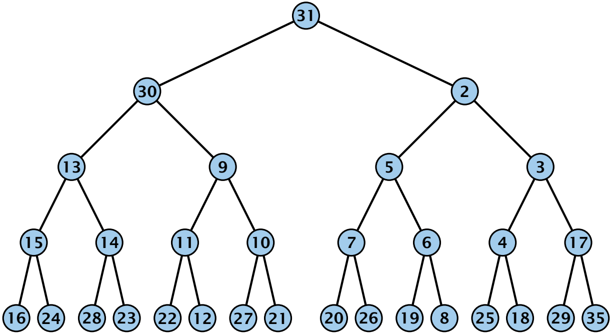

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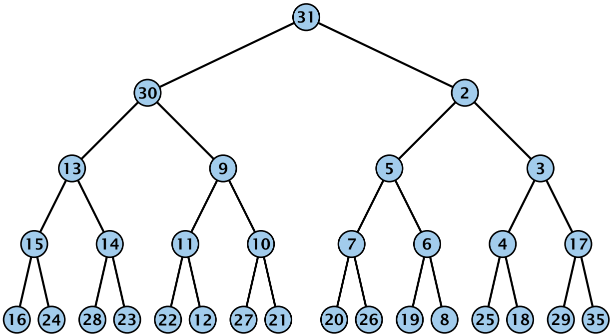

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Binary Heaps

Operations:

- **minimum()**: Return the root-element. Time $\mathcal{O}(1)$.
- **is-empty()**: Check whether root-pointer is null. Time $\mathcal{O}(1)$.
- **insert(k)**: Insert at x and bubble up. Time $\mathcal{O}(\log n)$.
- **delete(h)**: Swap with x and bubble up or sift-down. Time $\mathcal{O}(\log n)$.
- **build($x_1, \dots, x_n$)**: Insert elements arbitrarily; then do sift-down operations starting with the lowest layer in the tree. Time $\mathcal{O}(n)$.

Binary Heaps

The standard implementation of binary heaps is via arrays. Let $A[0, \dots, n-1]$ be an array

- The parent of i-th element is at position $\lfloor \frac{i-1}{2} \rfloor$.
- The left child of i-th element is at position $2i + 1$.
- The right child of i-th element is at position $2i + 2$.

Finding the successor of x is much easier than in the description on the previous slide. Simply increase or decrease x.

The resulting binary heap is not addressable. The elements don't maintain their positions and therefore there are no stable handles.

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