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How do you measure?

- Implementing and testing on representative inputs
  - How do you choose your inputs?
  - May be very time-consuming.
  - Very reliable results if done correctly.
  - Results only hold for a specific machine and for a specific set of inputs.

- Theoretical analysis in a specific model of computation.
  - Gives [illegible] like "this algorithm always runs in time $O(n^2)$".
  - Typically focuses on the [illegible].
  - Can give lower bounds like "any comparison-based sorting algorithm needs at least $\Omega(n \log n)$ comparisons in the worst case".

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Input length

The theoretical bounds are usually given by a function $f : \mathbb{N} \to \mathbb{N}$ that maps the input length to the running time (or storage space, comparisons, multiplications, program size etc.).

The input length may e.g. be

- the size of the input (number of bits)
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Suppose n numbers from the interval $\{1, \dots, N\}$ have to be sorted. In this case we usually say that the input length is n instead of e.g. $n \log N$, which would be the number of bits required to encode the input.

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1. Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM), . . .
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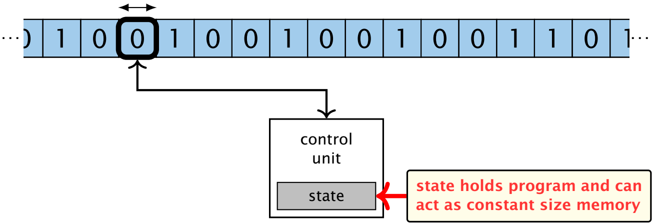
Turing Machine

- Very simple model of computation.
- Only the “current” memory location can be altered.
- Very good model for discussing computabiliy, or polynomial vs. exponential time.
- Some simple problems like recognizing whether input is of the form xx, where x is a string, have quadratic lower bound.

$\Rightarrow$ **Not a good model for developing efficient algorithms.**



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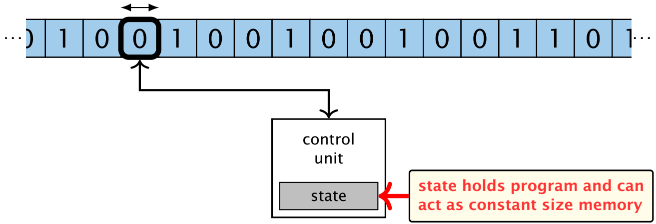

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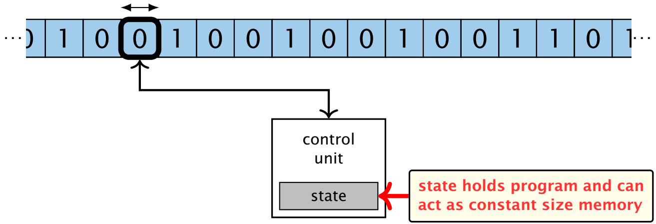

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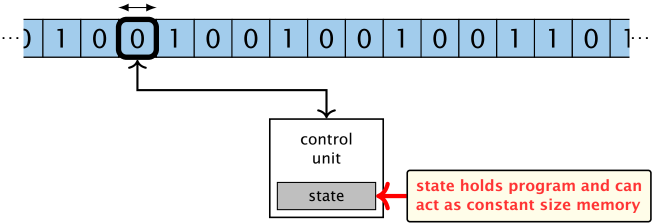

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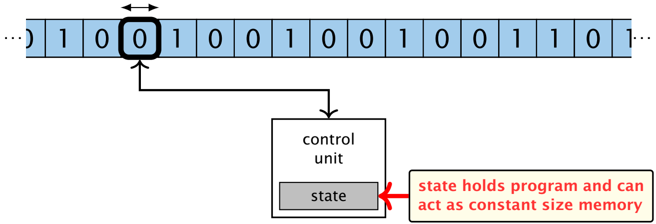
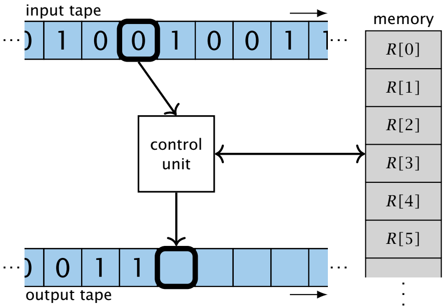


Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
- Indirect addressing.



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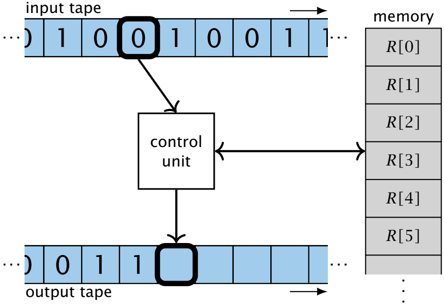

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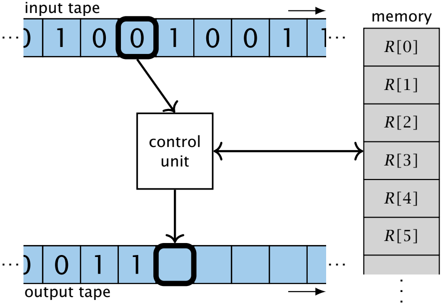

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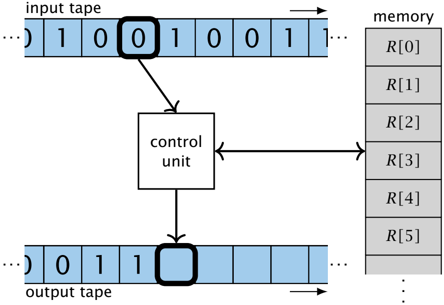

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- input operations (input tape $\rightarrow R[i]$)
  - READ i
- output operations ($R[i] \rightarrow$ output tape)
  - WRITE i
- register-register transfers
  - $R[j] := R[i]$
  - $R[j] := 4$
- **indirect** addressing
  - $R[j] := R[R[i]]$
    loads the content of the $R[i]$-th register into the j-th register
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    jumps to position x in the program;
    sets instruction counter to x;
    reads the next operation to perform from register $R[x]$
  - `jumpz` x $R[i]$
    jump to x if $R[i] = 0$
    if not the instruction counter is increased by 1;
  - `jumpi` i
    jump to $R[i]$ (indirect jump);
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  Every operation takes time 1.

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  The cost depends on the content of memory cells:

Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed 2^w, where usually $w = \log_2 n$.

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Example 2

Algorithm 1 RepeatedSquaring(n)

$r \leftarrow 2$;
for $i = 1 \rightarrow n$ **do**
    $r \leftarrow r^2$
return r

running time

space requirement

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- best-case complexity:

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  Usually easy to analyze, but not very meaningful.

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