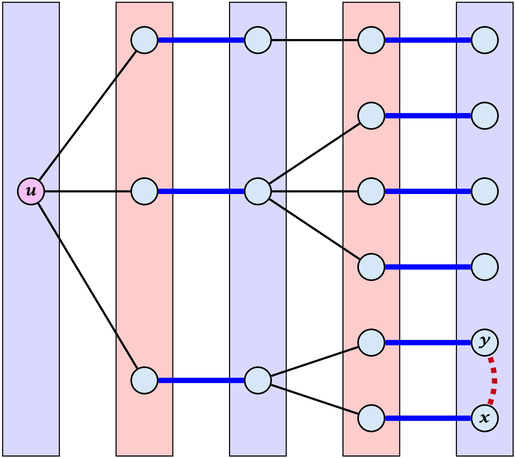
How to find an augmenting path?

Construct an alternating tree.



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y is already contained in T as an even vertex

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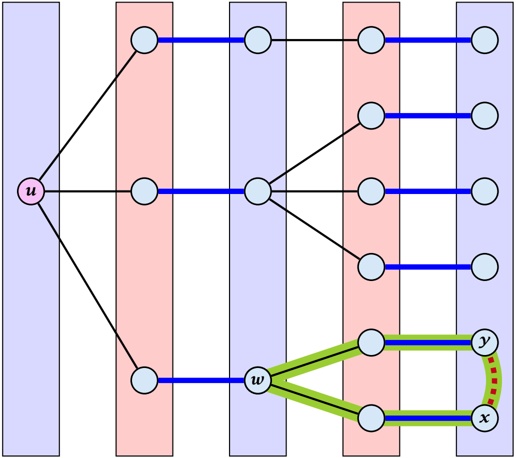

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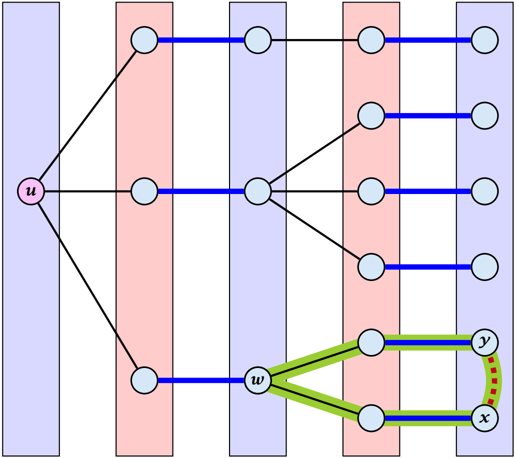

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The cycle $w \leftrightarrow y - x \leftrightarrow w$ is called a blossom. w is called the base of the blossom (even node!!!). The path u-w is called the stem of the blossom.

Flowers and Blossoms

Definition 1

A flower in a graph $G = (V, E)$ w.r.t. a matching M and a (free) root node r, is a subgraph with two components:

- A stem is an even length alternating path that starts at the root node r and terminates at some node w. We permit the possibility that $r = w$ (empty stem).
- A blossom is an odd length alternating cycle that starts and terminates at the terminal node w of a stem and has no other node in common with the stem. w is called the base of the blossom.

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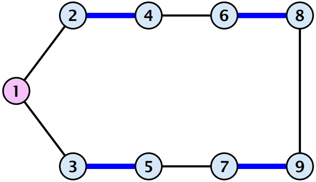

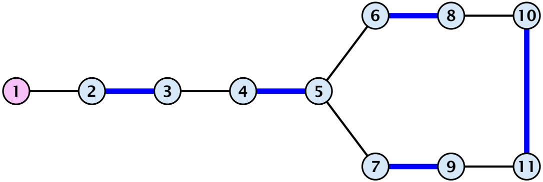

Flowers and Blossoms

Properties:

1. A stem spans $2\ell + 1$ nodes and contains ℓ matched edges for some integer $\ell \geq 0$.
2. A blossom spans $2k + 1$ nodes and contains k matched edges for some integer $k \geq 1$. The matched edges match all nodes of the blossom except the base.
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4. Every node x in the blossom (except its base) is reachable from the root (or from the base of the blossom) through two distinct alternating paths; one with even and one with odd length.
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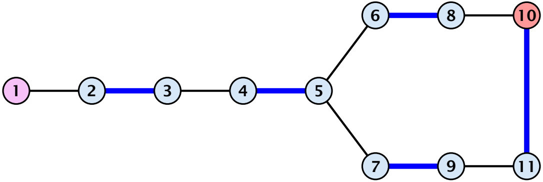

Shrinking Blossoms

When during the alternating tree construction we discover a blossom B we replace the graph G by $G' = G/B$, which is obtained from G by contracting the blossom B.

- Delete all vertices in B (and its incident edges) from G.
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- Edges of T that connect a node u not in B to a node in B become tree edges in T' connecting u to b.
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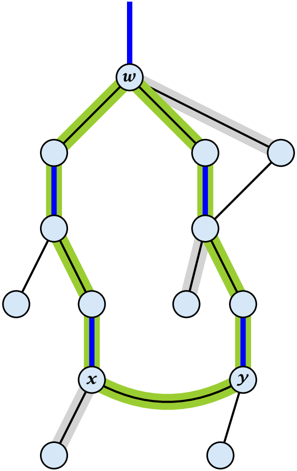

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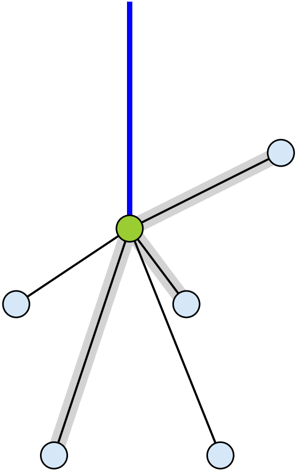
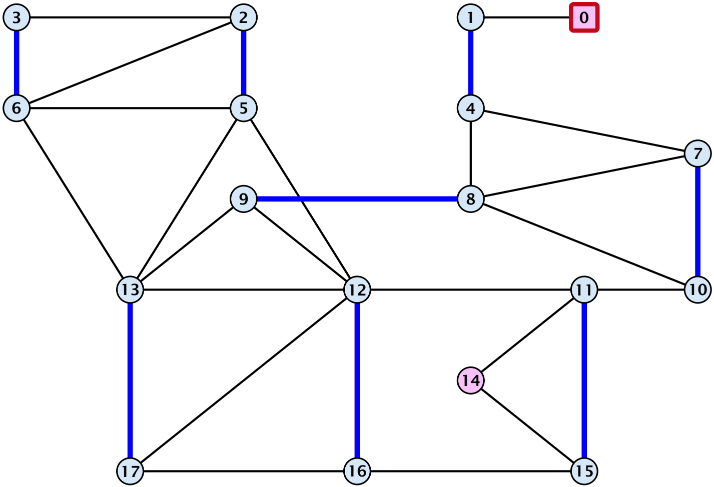


Example: Blossom Algorithm



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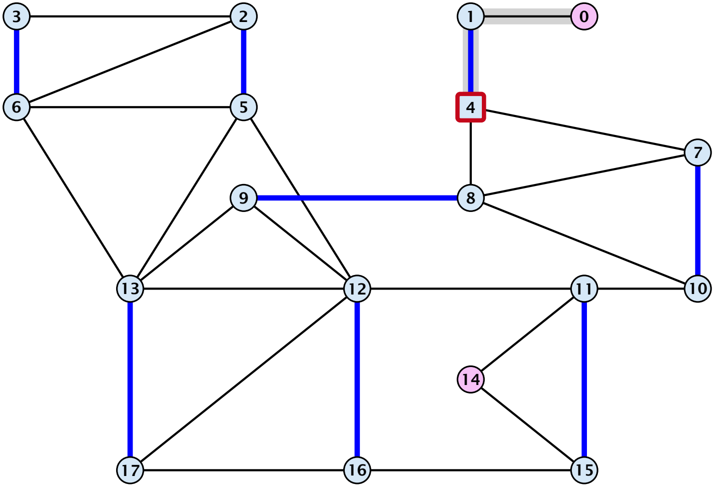

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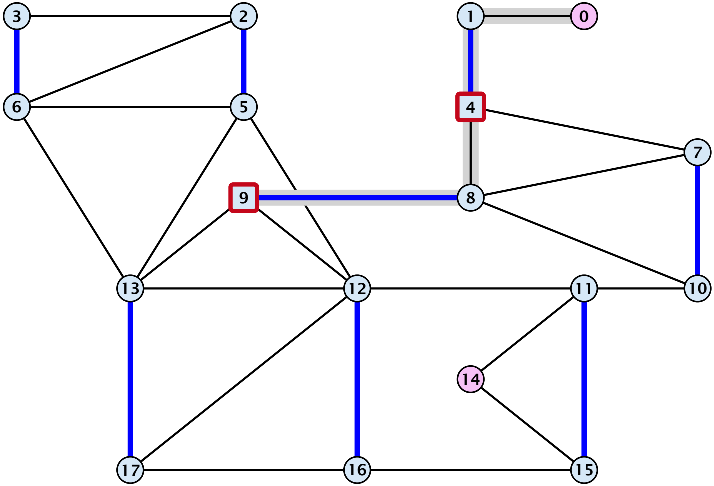

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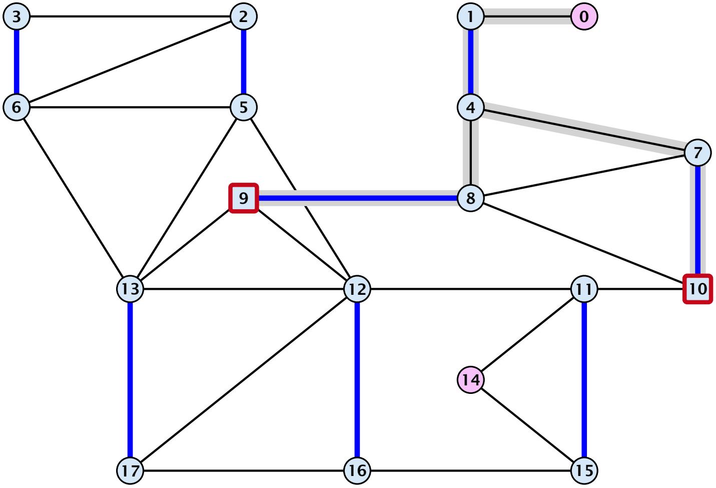

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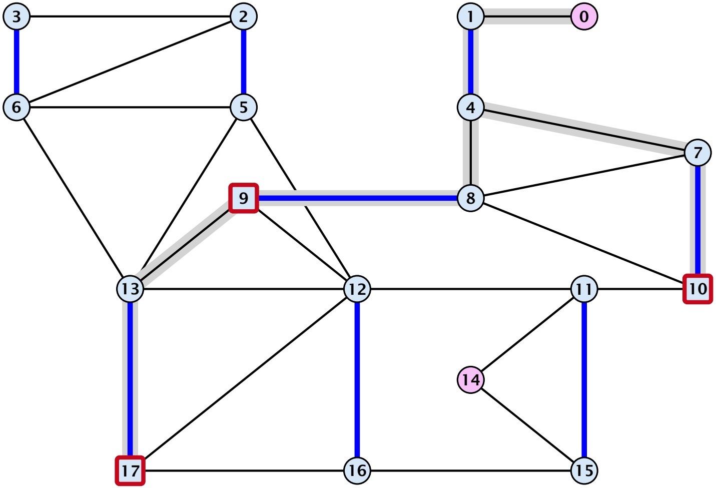

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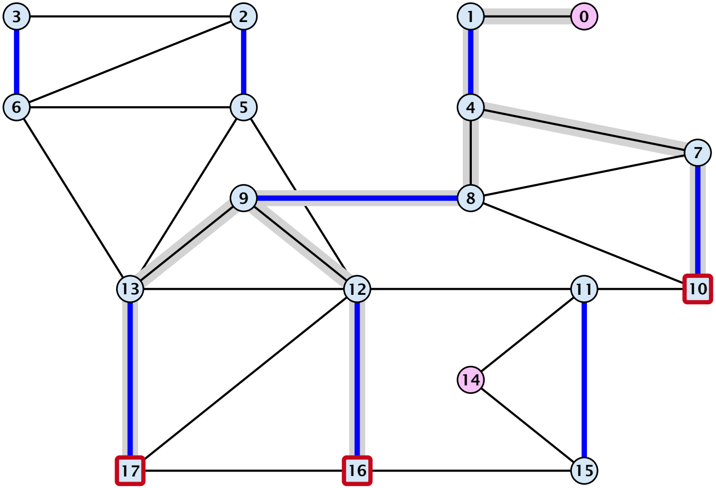

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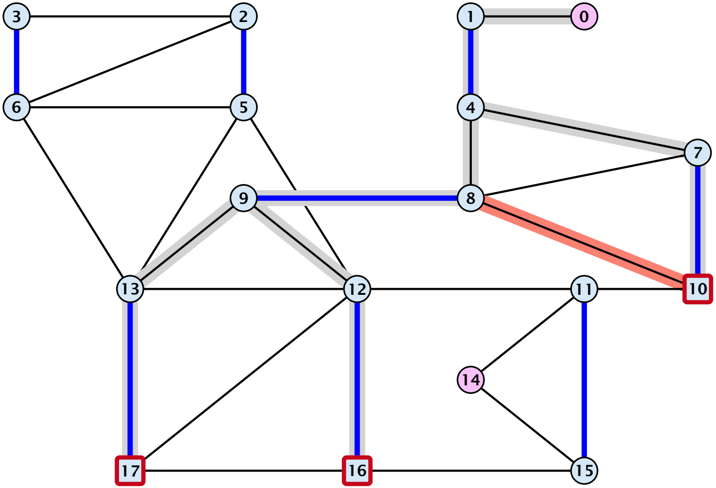

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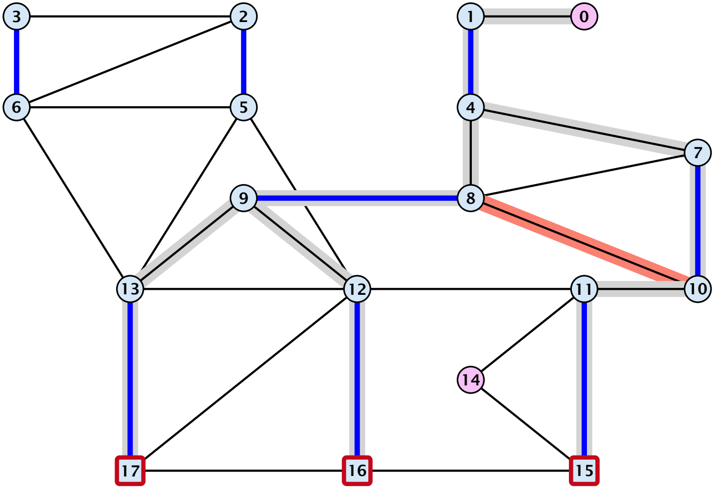

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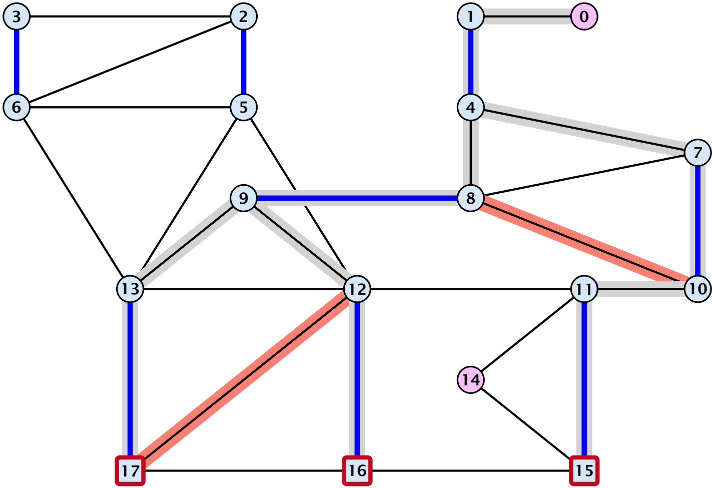

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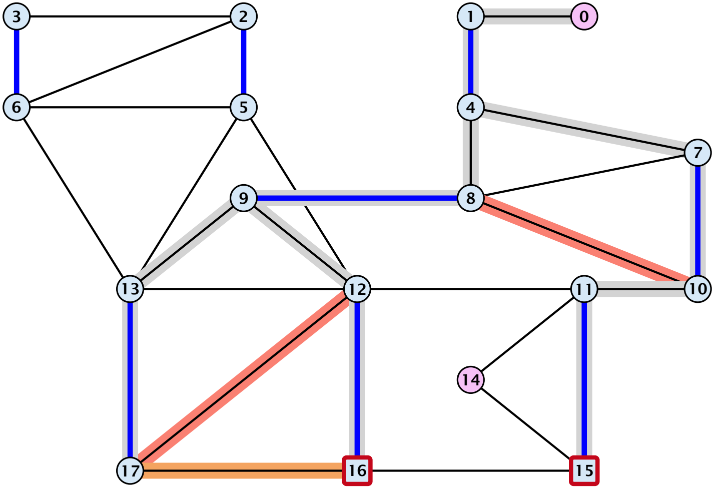

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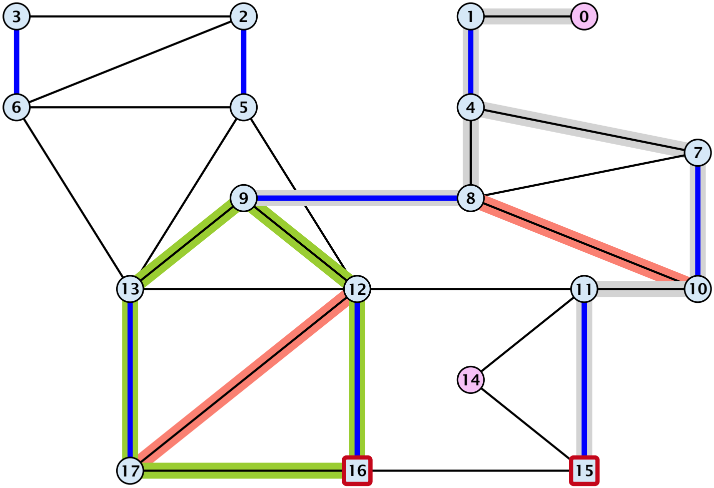

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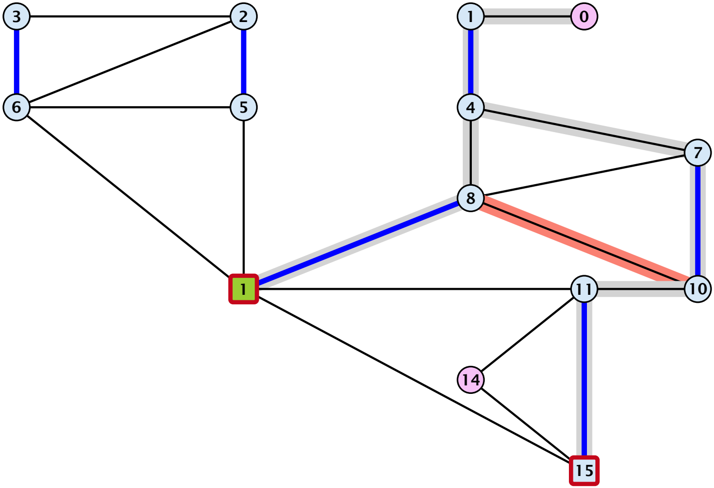

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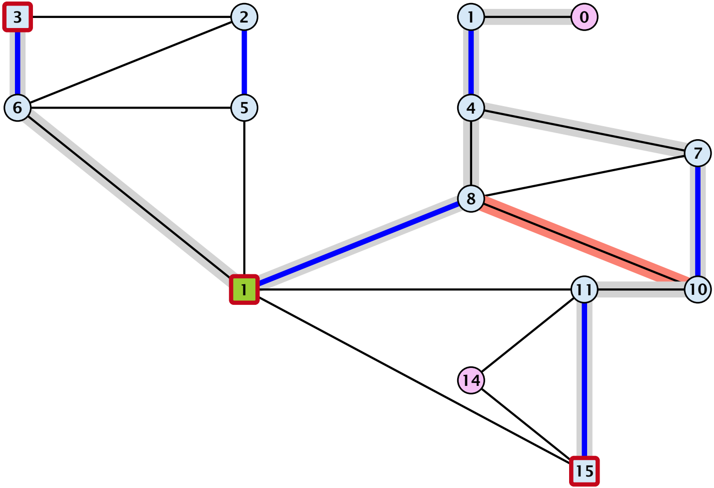

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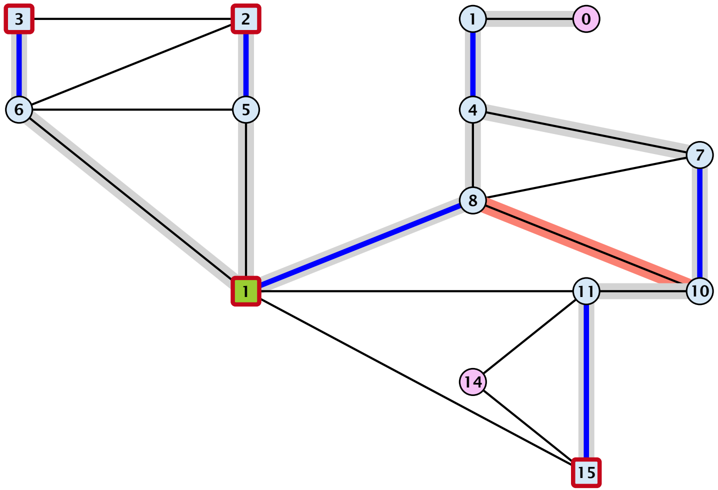

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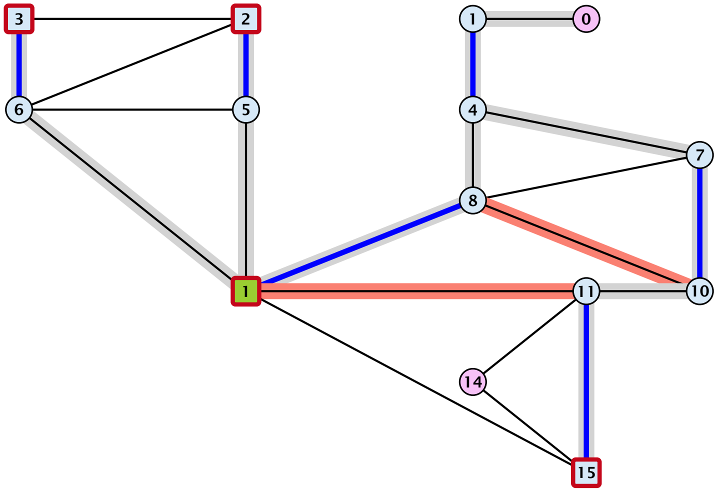

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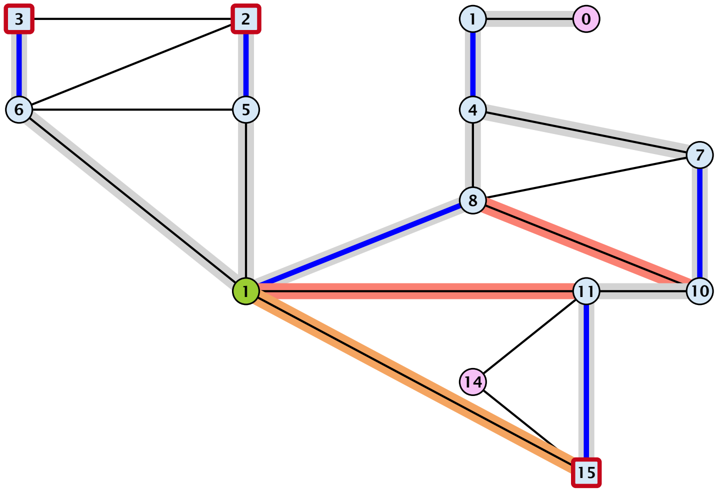

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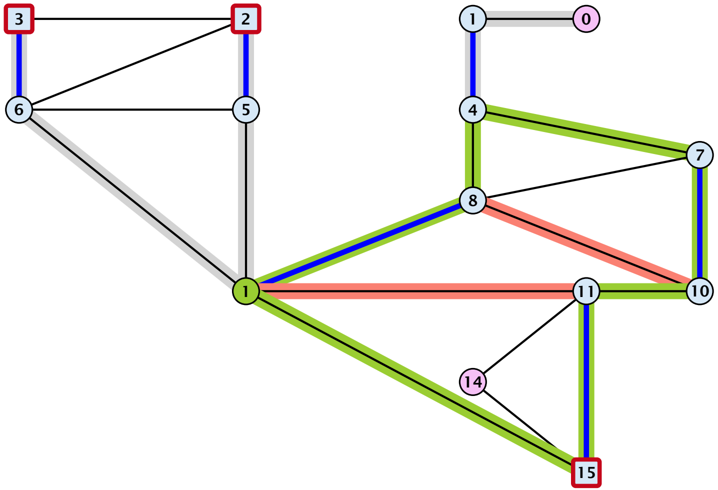

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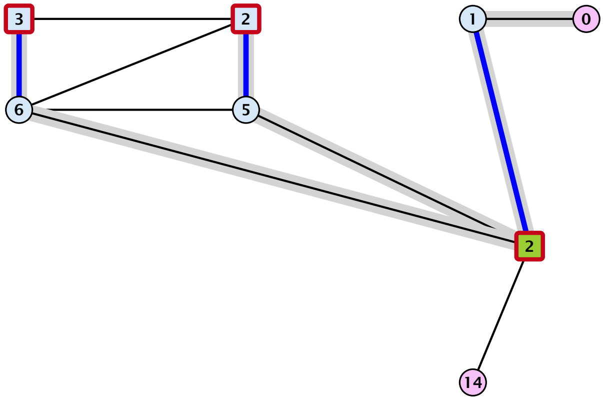

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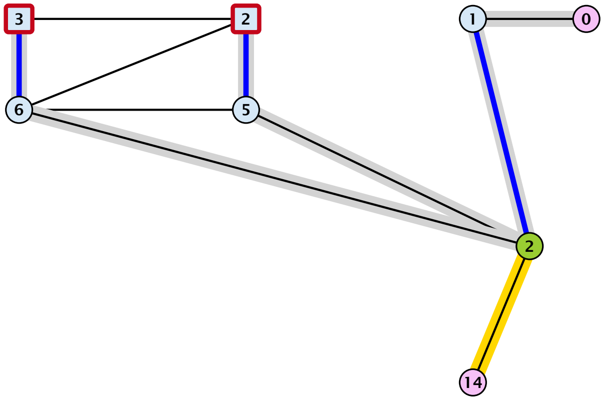

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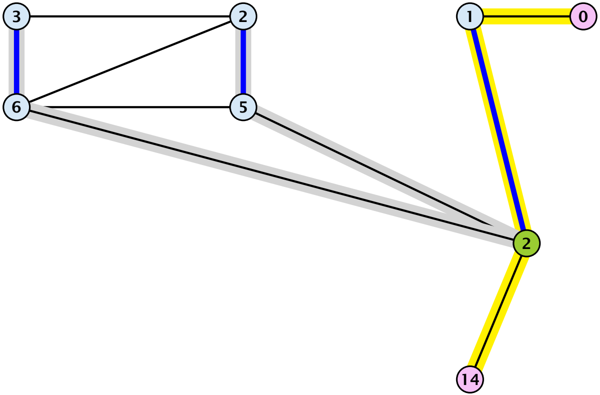

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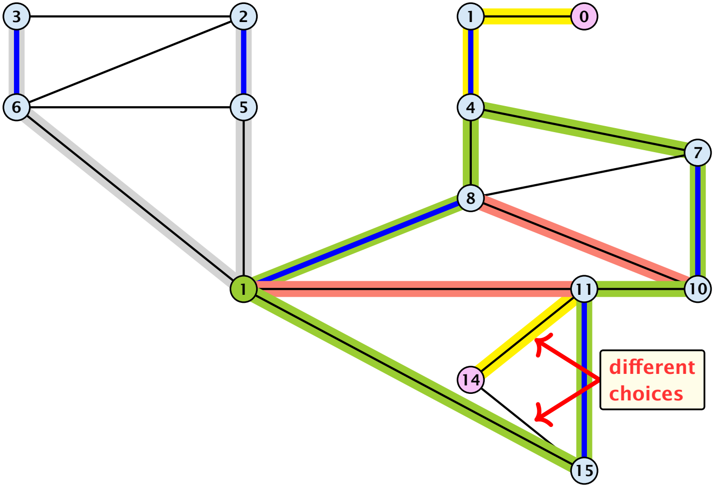

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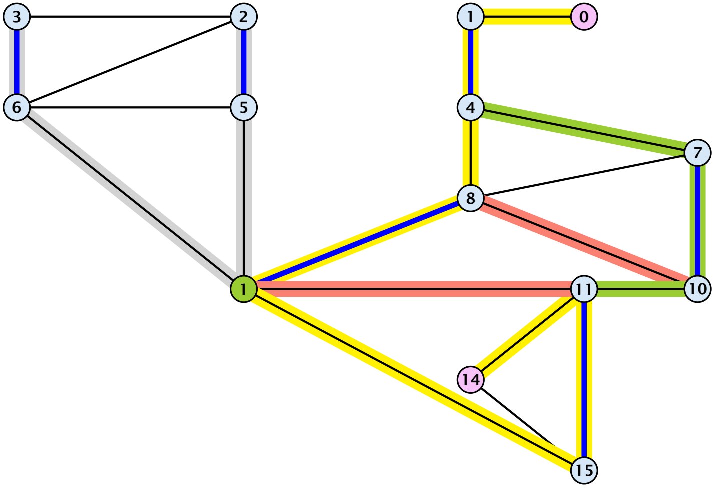

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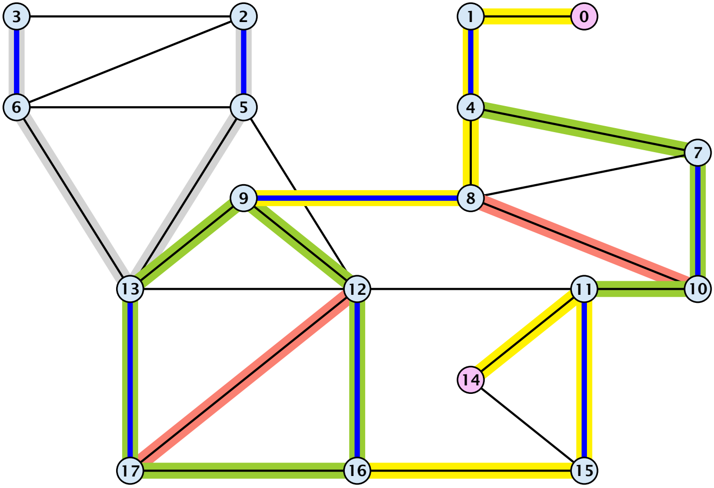

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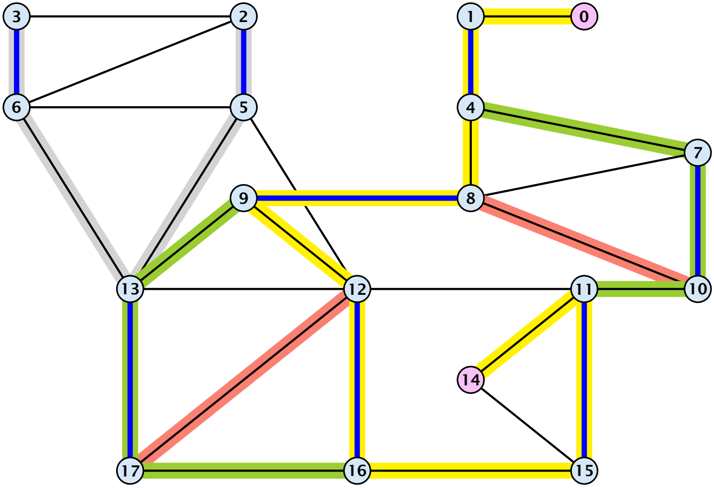

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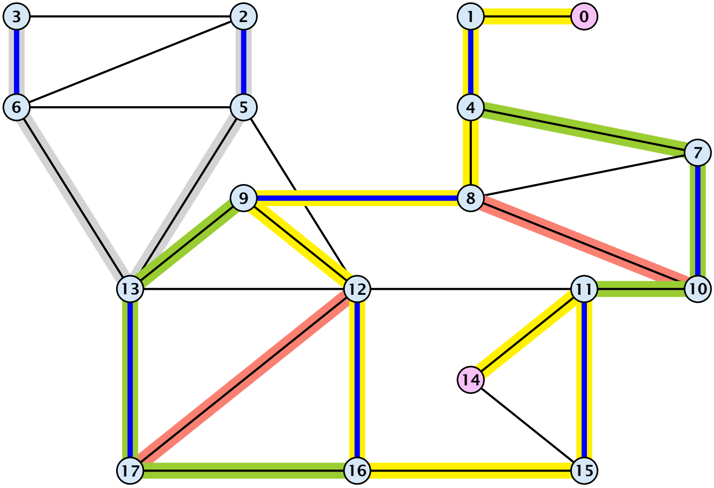

Correctness

Assume that in G we have a flower w.r.t. matching M. Let r be the root, B the blossom, and w the base. Let graph $G' = G/B$ with pseudonode b. Let M' be the matching in the contracted graph.

Lemma 2

If G' contains an augmenting path P' starting at r (or the pseudo-node containing r) w.r.t. the matching M' then G contains an augmenting path starting at r w.r.t. matching M.

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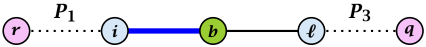

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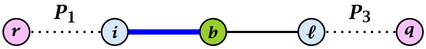

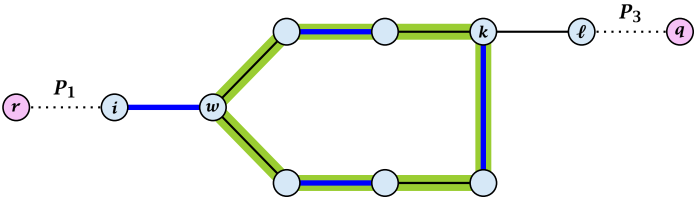

Correctness

- After the expansion ℓ must be incident to some node in the blossom. Let this node be k.
- If $k \neq w$ there is an alternating path P_2 from w to k that ends in a matching edge.
- $P_1 \circ (i, w) \circ P_2 \circ (k, \ell) \circ P_3$ is an alternating path.
- If $k = w$ then $P_1 \circ (i, w) \circ (w, \ell) \circ P_3$ is an alternating path.

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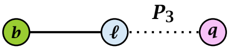

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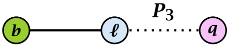

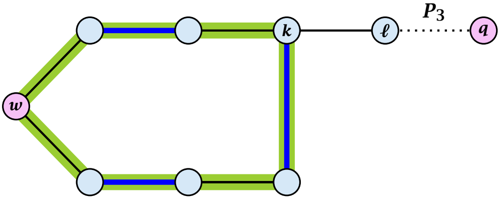

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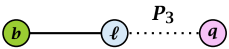

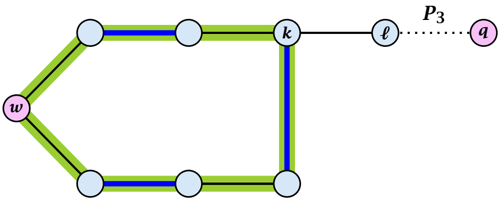

- The path $r \circ P_2 \circ (k, \ell) \circ P_3$ is an alternating path.

Correctness

Lemma 3
If G contains an augmenting path P from r to q w.r.t. matching M then G' contains an augmenting path from r (or the pseudo-node containing r) to q w.r.t. M'.

Correctness

Proof.

- If P does not contain a node from B there is nothing to prove.
- We can assume that r and q are the only free nodes in G.

Case 1: empty stem

Let i be the last node on the path P that is part of the blossom.

P is of the form $P_1 \circ (i, j) \circ P_2$, for some node j and (i, j) is unmatched.

$(b, j) \circ P_2$ is an augmenting path in the contracted network.

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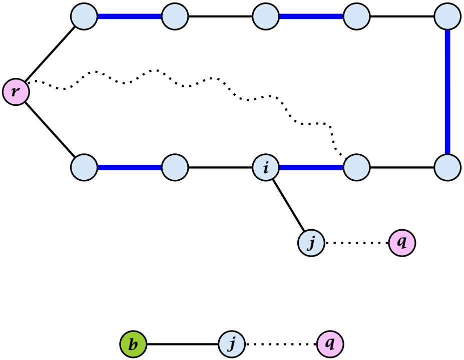
Illustration for Case 1:



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Let P_3 be alternating path from r to w; this exists because r and w are root and base of a blossom. Define $M_+ = M \oplus P_3$.

In M_+, r is matched and w is unmatched.

G must contain an augmenting path w.r.t. matching M_+, since M and M_+ have same cardinality.

This path must go between w and q as these are the only unmatched vertices w.r.t. M_+.

For M'_+ the blossom has an empty stem. Case 1 applies.

G' has an augmenting path w.r.t. M'_+. It must also have an augmenting path w.r.t. M', as both matchings have the same cardinality.

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This path must go between r and q.

Correctness

Case 2: non-empty stem

Let P_3 be alternating path from r to w; this exists because r and w are root and base of a blossom. Define $M_+ = M \oplus P_3$.

In M_+, r is matched and w is unmatched.

G must contain an augmenting path w.r.t. matching M_+, since M and M_+ have same cardinality.

This path must go between w and q as these are the only unmatched vertices w.r.t. M_+.

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This path must go between r and q.

Algorithm 24 search(r, *found*)

```
set Ā(i) ← A(i) for all nodes i
found ← false
unlabel all nodes;
give an even label to r and initialize list ← {r}
while list ≠ ∅ do
    delete a node i from list
    examine(i, found)
    if found = true then return
```

Search for an augmenting path starting at r.

Algorithm 24 search(r, *found*)

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unlabel all nodes;
give an even label to r and initialize list ← {r}
while list ≠ ∅ do
        delete a node i from list
        examine(i, found)
        if found = true then return
```

$A(i)$ contains neighbours of node i.

We create a copy $\bar{A}(i)$ so that we later can shrink blossoms.

Algorithm 24 search(r, *found*)

set $\bar{A}(i) \leftarrow A(i)$ for all nodes i

found $\leftarrow$ false

unlabel all nodes;

give an even label to r and initialize *list* $\leftarrow \{r\}$

while *list* $\neq \emptyset$ **do**

    delete a node i from *list*

    examine(i, *found*)

    if *found* $=$ true **then return**

found is just a Boolean that allows to abort the search process...

Algorithm 24 search(r, *found*)

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```

In the beginning no node is in the tree.

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found $\leftarrow$ false
unlabel all nodes;
give an even label to r and initialize *list* $\leftarrow \{r\}$
while *list* $\neq \emptyset$ **do**
    delete a node i from *list*
    examine(i, *found*)
    if *found* $=$ true **then return**

Put the root in the tree.

list could also be a set or a stack.

Algorithm 24 search(r, *found*)

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found $\leftarrow$ false
unlabel all nodes;
give an even label to r and initialize *list* $\leftarrow \{r\}$
while *list* $\neq \emptyset$ **do**
    delete a node i from *list*
    examine(i, *found*)
    if *found* $=$ true **then return**

As long as there are nodes with unexamined neighbours...

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while list ≠ ∅ do
    delete a node i from list
    examine(i, found)
    if found = true then return
```

...examine the next one

Algorithm 24 search(r, *found*)

set $\bar{A}(i) \leftarrow A(i)$ for all nodes i
found $\leftarrow$ false
unlabel all nodes;
give an even label to r and initialize *list* $\leftarrow \{r\}$
while *list* $\neq \emptyset$ **do**
    delete a node i from *list*
    examine(i, *found*)
    if *found* $=$ true **then return**

If you found augmenting path
abort and start from next root.

Algorithm 25 examine(i, *found*)

for all $j \in \bar{A}(i)$ **do**
  if j is even **then** contract(i, j) and **return**
  if j is unmatched **then**
    $q \leftarrow j$;
    $\mathrm{pred}(q) \leftarrow i$;
    $found \leftarrow \mathrm{true}$;
    return
  if j is matched and unlabeled **then**
    $\mathrm{pred}(j) \leftarrow i$;
    $\mathrm{pred}(\mathrm{mate}(j)) \leftarrow j$;
    add $\mathrm{mate}(j)$ to *list*

Examine the neighbours of a node i

Algorithm 25 examine(i, $found$)

```
for all j ∈ Ā(i) do
    if j is even then contract(i, j) and return
    if j is unmatched then
        q ← j;
        pred(q) ← i;
        found ← true;
        return
    if j is matched and unlabeled then
        pred(j) ← i;
        pred(mate(j)) ← j;
        add mate(j) to list
```

For all neighbours j do...

Algorithm 25 examine(i, *found*)

```
for all j ∈ Ā(i) do
    if j is even then contract(i, j) and return
    if j is unmatched then
        q ← j;
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        found ← true;
        return
    if j is matched and unlabeled then
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        add mate(j) to list
```

You have found a blossom...

Algorithm 25 examine(i, *found*)

for all $j \in \bar{A}(i)$ **do**
 if j is even **then** contract(i, j) and **return**
 if j is unmatched **then**
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 if j is matched and unlabeled **then**
  $\text{pred}(j) \leftarrow i$;
  $\text{pred}(\text{mate}(j)) \leftarrow j$;
  add $\text{mate}(j)$ to *list*

You have found a free node which gives you an augmenting path.

Algorithm 25 examine(i, *found*)

```
for all j ∈ Ā(i) do
    if j is even then contract(i, j) and return
    if j is unmatched then
        q ← j;
        pred(q) ← i;
        found ← true;
        return
    if j is matched and unlabeled then
        pred(j) ← i;
        pred(mate(j)) ← j;
        add mate(j) to list
```

If you find a matched node that is not in the tree you grow...

Algorithm 25 examine(i, *found*)

for all $j \in \bar{A}(i)$ **do**
    if j is even **then** contract(i, j) and **return**
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        $q \leftarrow j$;
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        return
    if j is matched and unlabeled **then**
        $\text{pred}(j) \leftarrow i$;
        $\text{pred}(\text{mate}(j)) \leftarrow j$;
        add $\text{mate}(j)$ to *list*

$\mathit{mate}(j)$ is a new node from which you can grow further.

Algorithm 26 contract(i, j)

trace pred-indices of i and j to identify a blossom B
create new node b and set $\bar{A}(b) \leftarrow \cup_{x \in B} \bar{A}(x)$
label b even and add to *list*
update $\bar{A}(j) \leftarrow \bar{A}(j) \cup \{b\}$ for each $j \in \bar{A}(b)$
form a circular double linked list of nodes in B
delete nodes in B from the graph

Contract blossom identified by nodes i and j

Algorithm 26 contract(i, j)

1: trace pred-indices of i and j to identify a blossom B
2: create new node b and set $\bar{A}(b) \leftarrow \cup_{x \in B} \bar{A}(x)$
3: label b even and add to *list*
4: update $\bar{A}(j) \leftarrow \bar{A}(j) \cup \{b\}$ for each $j \in \bar{A}(b)$
5: form a circular double linked list of nodes in B
6: delete nodes in B from the graph

Get all nodes of the blossom.

Time: $\mathcal{O}(m)$

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Identify all neighbours of b.

Time: $\mathcal{O}(m)$ (how?)

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b will be an even node, and it has unexamined neighbours.

Algorithm 26 contract(i, j)

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update $\bar{A}(j) \leftarrow \bar{A}(j) \cup \{b\}$ for each $j \in \bar{A}(b)$

form a circular double linked list of nodes in B

delete nodes in B from the graph

Every node that was adjacent to a node in B is now adjacent to b

Algorithm 26 contract(i, j)

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Only for making a blossom expansion easier.

Algorithm 26 contract(i, j)

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5: form a circular double linked list of nodes in B
6: delete nodes in B from the graph

Only delete links from nodes not in B to B.

When expanding the blossom again we can recreate these links in time $\mathcal{O}(m)$.

Analysis

- A contraction operation can be performed in time $\mathcal{O}(m)$. Note, that any graph created will have at most m edges.
- The time between two contraction-operation is basically a BFS/DFS on a graph. Hence takes time $\mathcal{O}(m)$.
- There are at most n contractions as each contraction reduces the number of vertices.
- The expansion can trivially be done in the same time as needed for all contractions.
- An augmentation requires time $\mathcal{O}(n)$. There are at most n of them.
- In total the running time is at most

$$n \cdot (\mathcal{O}(mn) + \mathcal{O}(n)) = \mathcal{O}(mn^2) \ .$$

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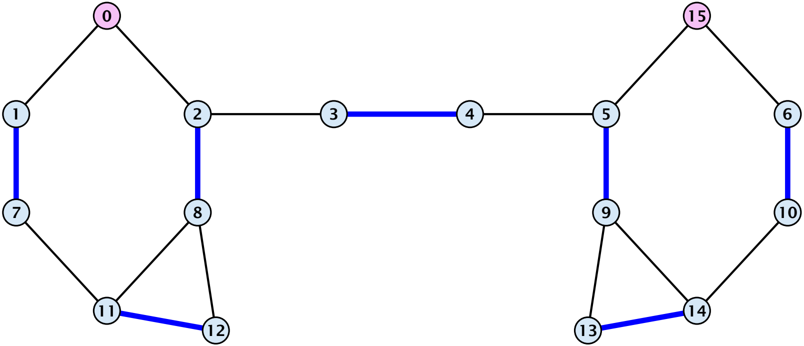

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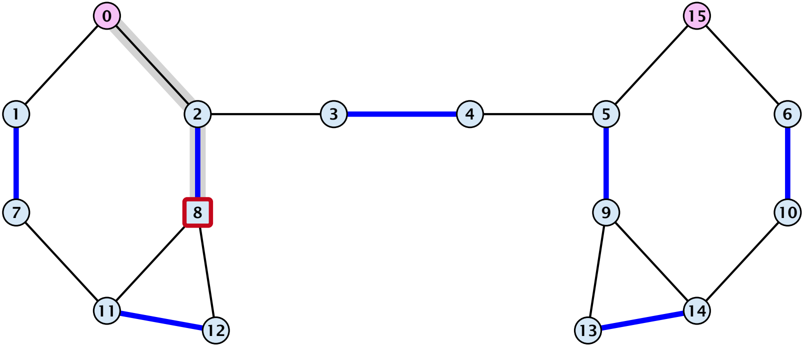

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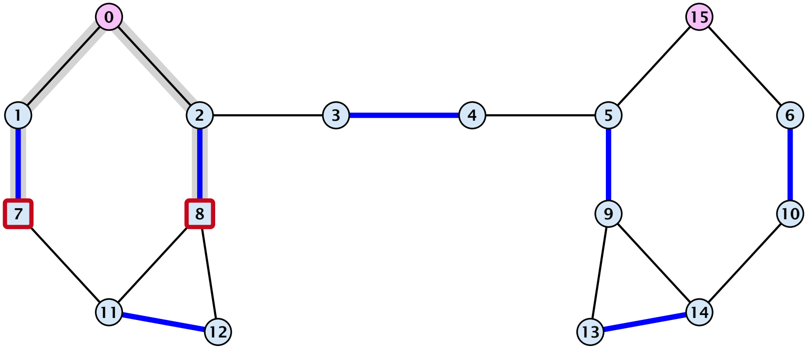

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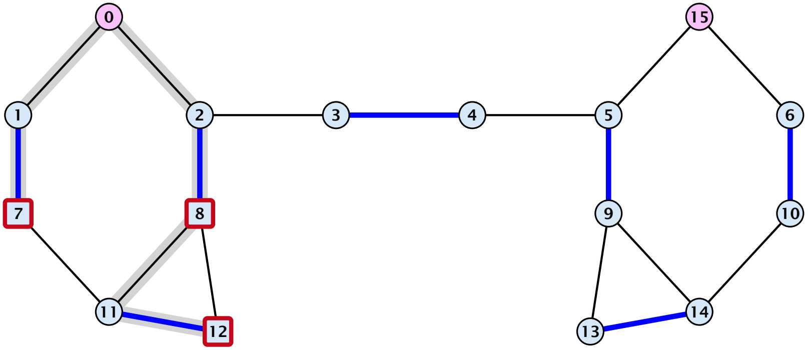

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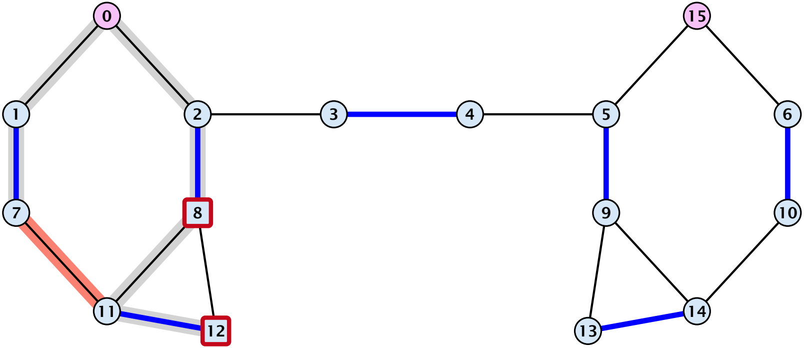

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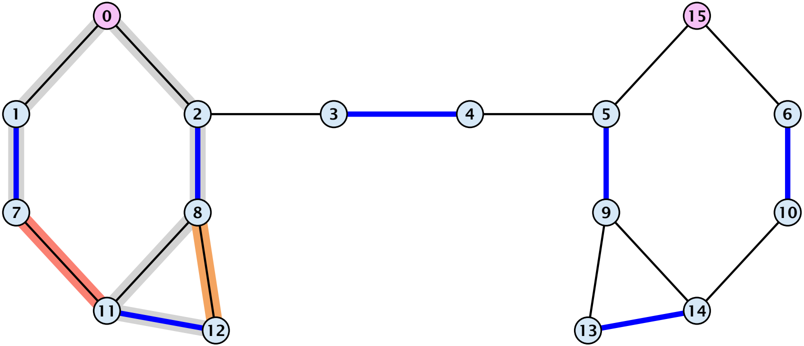

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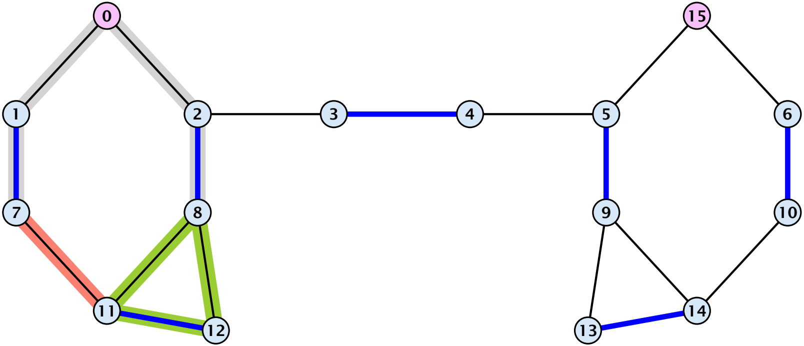

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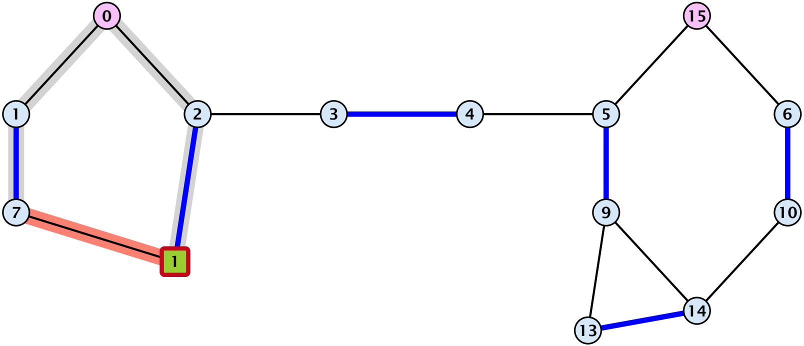

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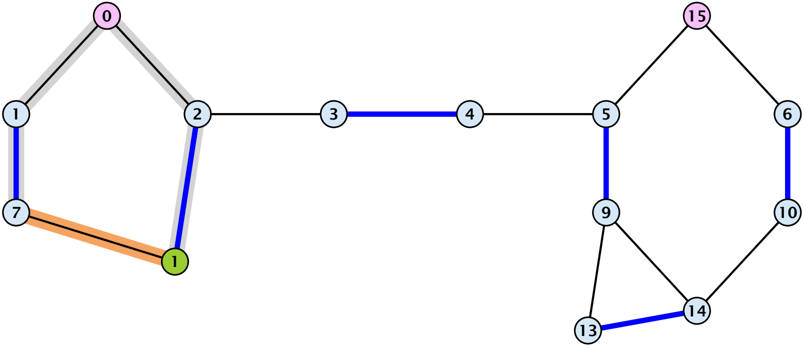

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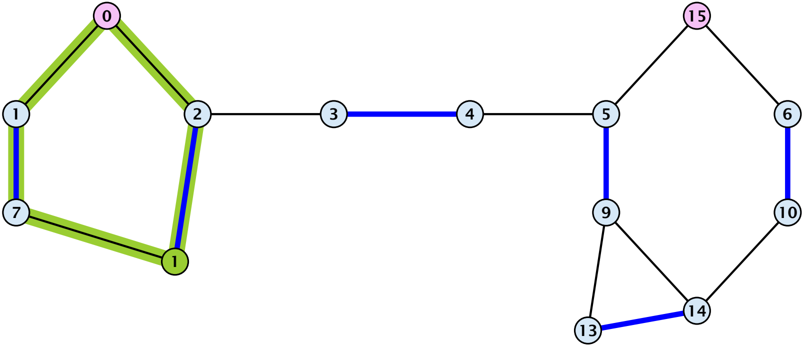

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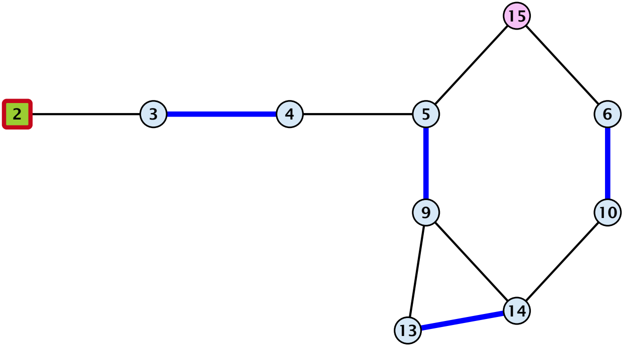

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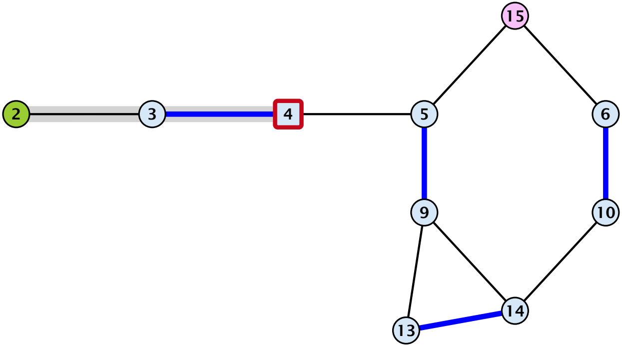

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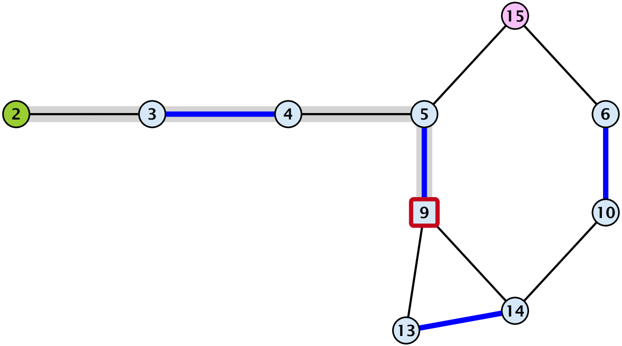

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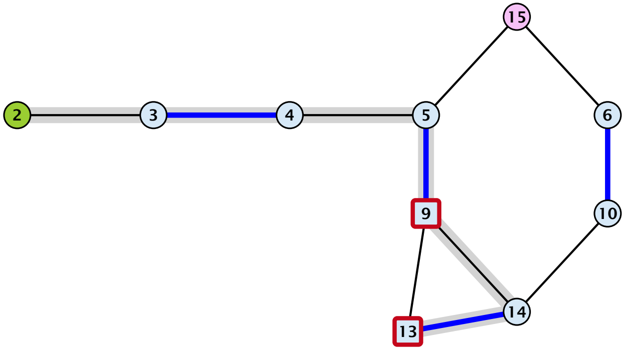

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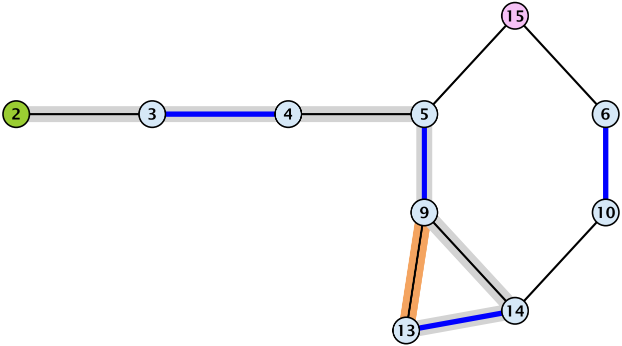

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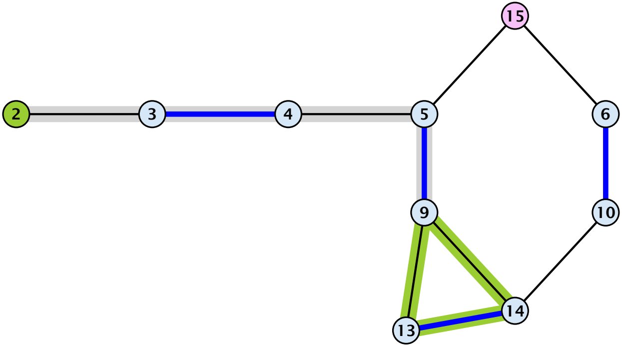

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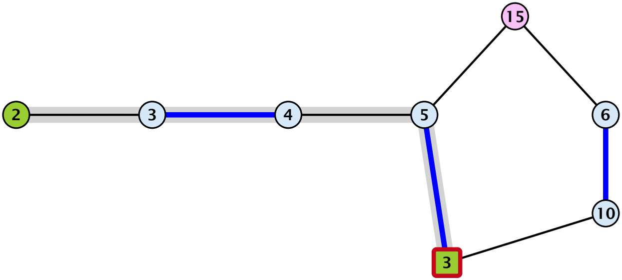

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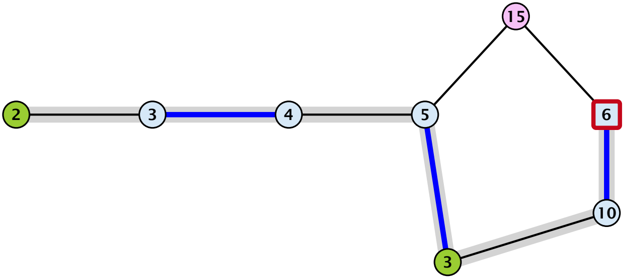

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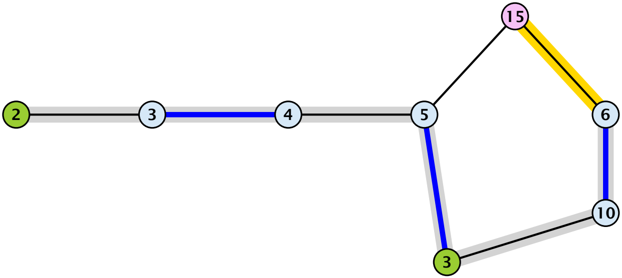

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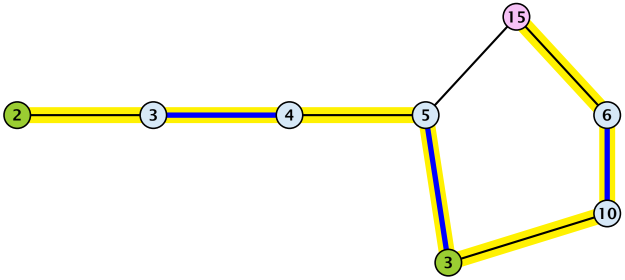

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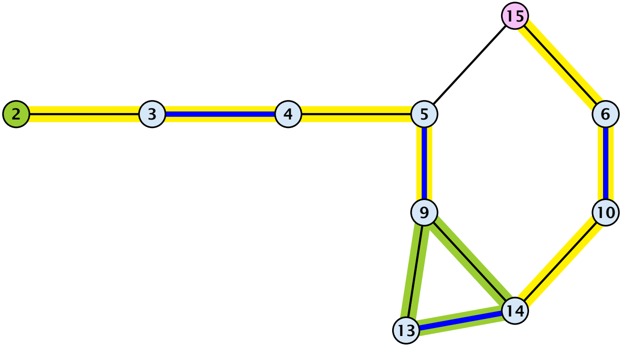

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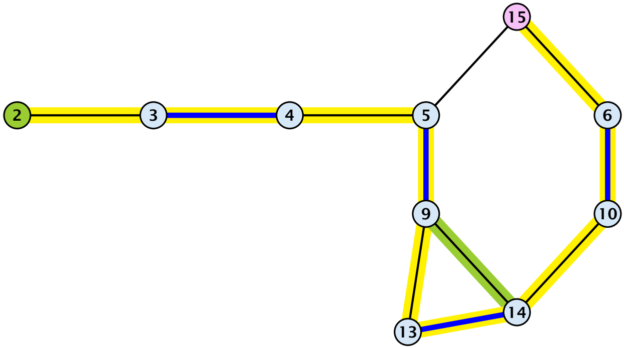

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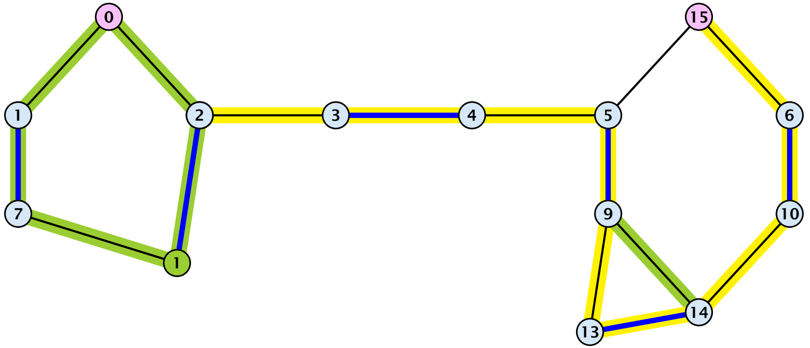

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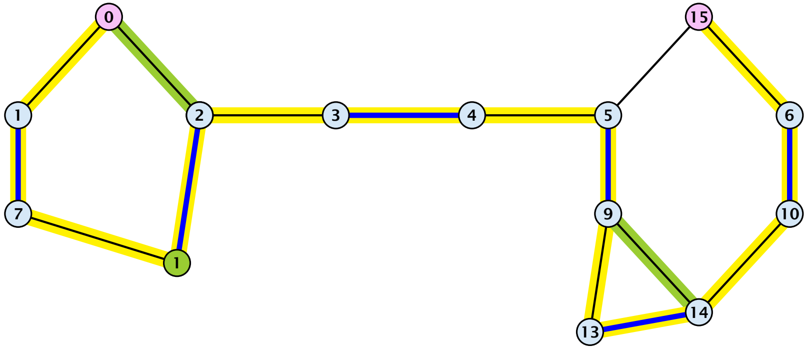

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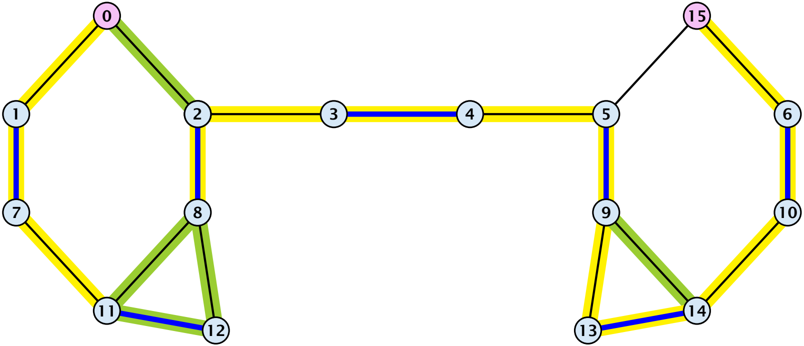
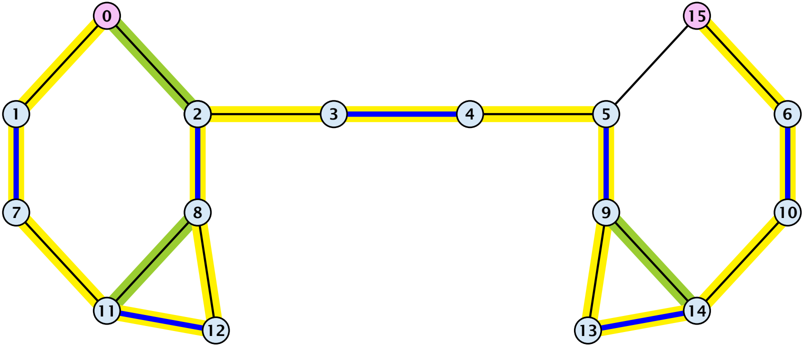


Example: Blossom Algorithm



Example: Blossom Algorithm