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Randomized Algorithms

Exercise Sheet 8

Due: December 14, 2015
at 10:15, in class

Exercise 8.1 (10 points)
Let X be a non-negative integer-valued random variable with positive expectation. Prove that

$$\frac{E[X]^2}{E[X^2]} \leq P[X \neq 0] \leq E[X].$$

Exercise 8.2 (10 points)
The use of the variance of a random variable in bounding its deviation from its expectation is called the *second moment method*.

In an analogous way, we can speak of the kth moment method. Given is a random variable Y with expectation μ_Y. Let k be even and we define $\mu_Y^k = E[(Y - \mu_Y)^k]$. Let us assume that we are dealing with a variable Y for which μ_Y^k exists. Show that

$$P\left[|Y - \mu_Y| \geq t\sqrt[k]{\mu_Y^k}\right] \leq \frac{1}{t^k}.$$

Exercise 8.3 (10 points)
We throw m balls into n bins independently and uniformly at random. Use Markov's and Chebyshev's inequalities in order to compute upper bounds on the probability that a bin contains at least k balls.

Compare these bounds when $m = n$.

Exercise 8.4 (10 points)
Your friendly neighbourhood grocery store knows that you are tired of getting coupons that you already have. So they have a special offer for you! There are n coupons to collect, but now at every time step they select, uniformly at random, k different coupons and then you can choose one to keep. You want to collect all n coupons.

Give a bound (as a function of k and n) on the expected number of coupons you select in order to have collected each coupon.