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Randomized Algorithms

Exercise Sheet 4

Due: November 16, 2015
at 10:15, in class

Exercise 4.1 (10 points)
Show that **ZPP** = **RP** $\cap$ co-**RP**.

Exercise 4.2 (10 points)
Show that **RP** $\subseteq$ **BPP** $\subseteq$ **PP**.

Exercise 4.3 (10 points)
Consider a uniform tree in which the root and every internal node have exactly 3 children. Every leaf is at distance h from the root (h is called the height of the tree) and it is associated with a boolean value 0 or 1. The value of a non-leaf node is the value of the majority of its children. The evaluation problem is to determine the value of the root.

(a) Show that for any deterministic algorithm A, there is an instance such that A will have to read all $n = 3^h$ leaves in order to evaluate the tree correctly.

(b) Consider the recursive randomized algorithm which evaluates two randomly selected subtrees of the root and if their values disagree, it evaluates the third subtree. Show that the expected number of leaves read by the algorithm is $O\left(n^{0.9}\right)$.

Exercise 4.4 (10 points)
Consider a 2-player zero-sum game specified by the following payoff matrix:

$$M = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$$

(a) Verify that there are no optimal pure strategies.

(b) Compute optimal mixed strategies for both players.

(c) What is the optimal strategy for each player assuming he knows the mixed strategy of his opponent?