Prof. Dr. Susanne Albers
Dr. Suzanne van der Ster
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Institut für Informatik
Technische Universität München

Fall Semester
January 18, 2016

Randomized Algorithms

Exercise Sheet 12

Due: January 25, 2016
at 10:15, in class

Exercise 12.1 (10 points)
Consider a complete graph K_n on n vertices and consider coloring the edges of that graph with two colors.

(a) Prove that, for every integer $n \geq 4$, there exists a coloring of the edges of K_n with two colors such that the total number of monochromatic K_4 cliques is at most $\binom{n}{4}\left(\frac{1}{2}\right)^5$.
Hint: Use an expectation argument.

(b) Give a randomized algorithm for finding a coloring with at most $\binom{n}{4}\left(\frac{1}{2}\right)^5$ monochromatic K_4 cliques with an expected running time polynomial in n.

Exercise 12.2 (10 points)
Consider an instance of SAT with m clauses, where every clause has exactly k literals.

(a) Give a Las Vegas algorithm that finds an assignment satisfying at least $m(1-2^{-k})$ clauses and analyze its expected running time.

(b) Give a derandomization of the randomized algorithm using the method of conditional expectations.

Exercise 12.3 (10 points)
Given an n-vertex undirected graph $G = (V, E)$, consider the following method of generating an independent set. Given a permutation σ of the vertices, define a subset $S(\sigma)$ of the vertices as follows: for each vertex i, $i \in S(\sigma)$ if and only if no neighbor j of i precedes i in the permutation σ.

(a) Show that each $S(\sigma)$ is an independent set in G.

(b) Suggest a natural randomized algorithm to produce σ for which you can show that the expected cardinality of $S(\sigma)$ is

$$\sum_{i=1}^{n} \frac{1}{d_i + 1}$$

where d_i denotes the degree of vertex i.

(c) Prove that G has an independent set of size at least $\sum_{i=1}^{n} 1/(d_i + 1)$.

Exercise 12.4 (10 points)
We have shown using the probabilistic method that, if a graph G has n nodes and m edges, then there exists a partition of the n nodes into sets A and B such that at least $m/2$ edges cross the partition. Improve this result slightly: show that there exists a partition such that at least $mn/(2n-1)$ edges cross the partition.