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Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Suppose that a flow network has some infinite capacity edges but no infinite capacity paths from the source to the sink. Let F denote the set of edges with finite capacities. Show that we can replace the capacity of each infinite capacity edge by the finite number $x = \sum_{(i,j)\in F} c(i,j)$ without affecting the maximum flow value.

Question 2 (10 Points)

The edge connectivity of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.

Question 3 (10 Points)

Consider a 0-1 matrix A with n rows and m columns. We refer to a row or a column of the matrix A as a *line*. We say that a set of 1's in the matrix A is *independent* if no two of them appear in the same line. We also say that a set of lines in the matrix is a *cover* of A if they cover all the 1's in the matrix. Using the max-flow min-cut theorem, show that the maximum number of independent 1's equals the minimum number of lines in a cover.

Question 4 (10 Points)

A town has r residents $R_1, \ldots, R_r$, q clubs $C_1, \ldots, C_q$, and p political parties $P_1, \ldots, P_p$. Each resident is a member of at least one club and belongs to exactly one political party. Each club must nominate one of its members to represent it on the town's governing council so that the number of council members belonging to the political party P_k is at most u_k. Using maxflows, find out whether it is possible for clubs nominate members in such a way.