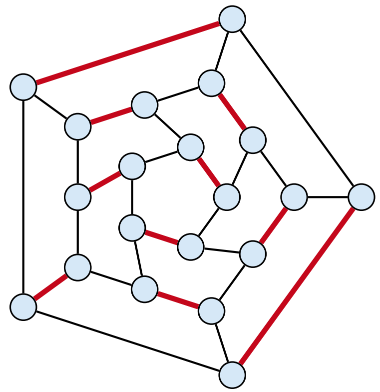
Part V

Matchings

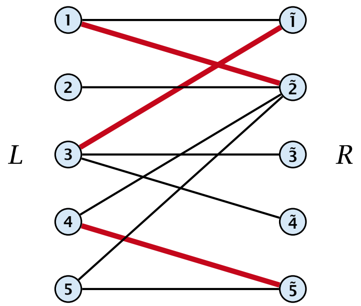
Matching

- Input: undirected graph $G = (V, E)$.
- $M \subseteq E$ is a matching if each node appears in at most one edge in M.
- Maximum Matching: find a matching of maximum cardinality



Bipartite Matching

- Input: undirected, bipartite graph $G = (L \uplus R, E)$.
- $M \subseteq E$ is a matching if each node appears in at most one edge in M.
- Maximum Matching: find a matching of maximum cardinality



Bipartite Matching

- Input: undirected, bipartite graph $G = (L \uplus R, E)$.
- $M \subseteq E$ is a matching if each node appears in at most one edge in M.
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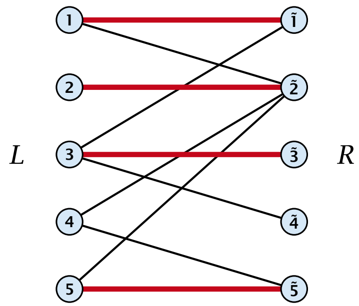
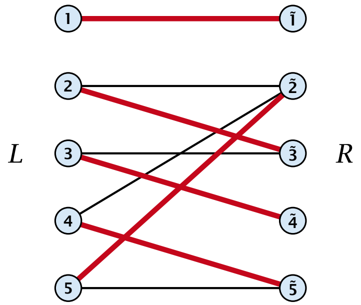


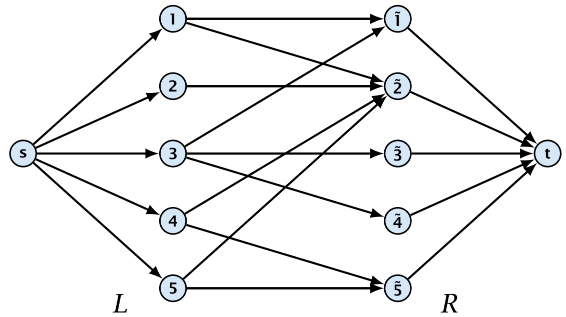
Bipartite Matching

- A matching M is perfect if it is of cardinality $|M| = |V|/2$.
- For a bipartite graph $G = (L \uplus R, E)$ this means $|M| = |L| = |R| = n$.



17 Bipartite Matching via Flows

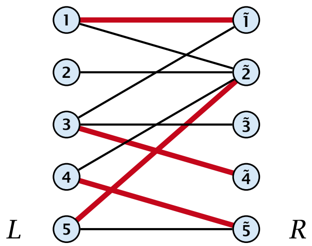
- Input: undirected, bipartite graph $G = (L \uplus R \uplus \{s,t\}, E')$.
- Direct all edges from L to R.
- Add source s and connect it to all nodes on the left.
- Add t and connect all nodes on the right to t.
- All edges have unit capacity.



Proof

Max cardinality matching in $G \leq$ value of maxflow in G'

- Given a maximum matching M of cardinality k.
- Consider flow f that sends one unit along each of k paths.
- f is a flow and has cardinality k.



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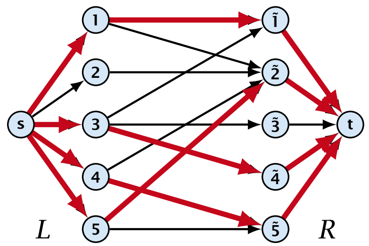
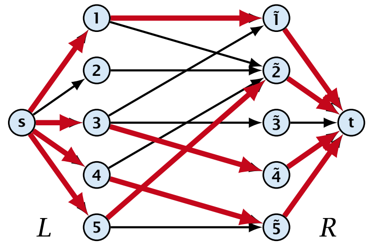


G'

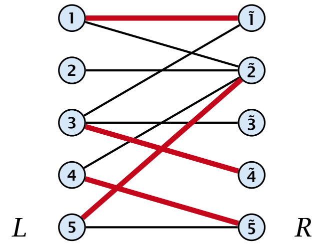
Proof

Max cardinality matching in $G \geq$ value of maxflow in G'

- Let f be a maxflow in G' of value k
- Integrality theorem $\Rightarrow$ k integral; we can assume f is 0/1.
- Consider M= set of edges from L to R with $f(e) = 1$.
- Each node in L and R participates in at most one edge in M.
- $|M| = k$, as the flow must use at least k middle edges.



G'



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