Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
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Assignment 13
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Efficient Algorithms and Data Structures I

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*Deadline: February 6, 2017, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (5 Points)

Let f be a flow in a network, and let α be a real number. The scalar flow product, denoted $\alpha \cdot f$, is a function from $V \times V$ to R defined by

$$(\alpha \cdot f)(u, v) = \alpha \cdot f(u, v)$$

Prove that the feasible flows in a network form a convex set. That is, show that if f_1 and f_2 are flows, then so is $\alpha f_1 + (1 - \alpha) f_2$ for $0 \leq \alpha \leq 1$.

Homework 2 (5 Points)

We say that a bipartite graph $G = (V, E)$, where $V = L \cup R$, is d-regular if every vertex $v \in V$ has degree exactly d. Every d-regular bipartite graph has $|L| = |R|$. Prove that every d-regular bipartite graph has a matching of cardinality $|L|$ by arguing that a minimum cut of the corresponding flow network has capacity $|L|$.

Homework 3 (5 Points)

During his daily stroll through the FMI building in Garching, the ghost Ambrosius finds a mysterious inscription on the whiteboard in some scientist's office. It is an $m \times n$ matrix, with $m > n$ filled with integers from $\{1, \ldots, m\}$. Ambrosius notes that no row or column contains the same number twice.

Ambrosius wants to prank the scientist by extending the matrix to an $m \times m$ matrix, effectively adding $m - n$ columns. To make his prank even funnier, he plans to fill the new columns with integers from $\{1, \ldots, m\}$ such that the new matrix still has no row or column containing the same number twice.

Show that Ambrosius' plan can succeed!

Hint: Use Hall's theorem!

Homework 4 (5 Points)

The dean needs to assign a group of n faculty members to be chairs on n committees. Each faculty member proposes, in decreasing order of preference, a list of three committees that he or she would like to chair. We want to determine whether there exists a *satisfiable assignment* (one that assigns the faculty to the committees so that each faculty member obtains a job on his or her list). If some satisfiable assignment is possible, we want to find the assignment that maximizes the number of faculty with their most preferred committee chair, and further, among such assignments, the assignment that maximizes the number of faculty with their second most preferred committee chair. Show how to solve this problem by solving a single assignment problem.

Bonus Homework 1 (5 Bonus Points)

If you have any open questions about the course contents that you want to discuss with the tutor, please tell them via email 48 hours before the tutorial.

You may earn bonus points for asking good questions.

Tutorial Exercise 1

Discuss open questions with you tutor!

> If you find that you're spending almost all your time on theory, start turning some attention to practical things; it will improve your theories. If you find that you're spending almost all your time on practice, start turning some attention to theoretical things; it will improve your practice.
>
>   - D. E. Knuth