Technische Universität München
Fakultät für Informatik
Lehrstuhl für Theoretische Informatik
Prof. Dr. Harald Räcke
Richard Stotz, Dennis Kraft

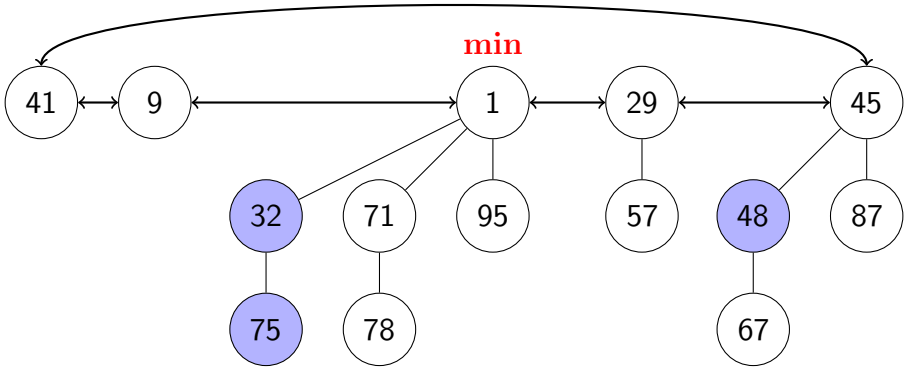
Winter term 2016 / 2017
Assignment 10
January 9, 2017

Efficient Algorithms and Data Structures I

*Deadline: January 16, 2017, 10:15 am in the **Efficient Algorithms** mailbox.*

Homework 1 (5 Points)

Perform the following operations sequentially on the Fibonacci Heap shown below so that it remains a Fibonacci Heap. Show what the heap looks like after each operation (always carry out each operation on the result of the previous operation).
Nodes that are marked appear in blue.



1. Insert(38)
2. DeleteMin()
3. DecreaseKey(87,60)
4. DecreaseKey(67,6)

Homework 2 (5 Points)

Consider Union-Find with trees without path compression. Give a sequence of m MAKESET, UNION and FIND operations, n of which are MAKESET operations, that take $\Omega(m \log n)$ operations. Assume that $m \geq 4n$.

Homework 3 (5 Points)

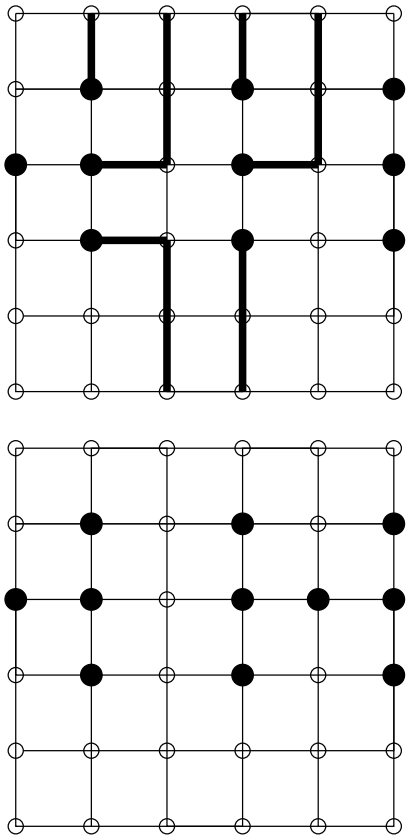
Show that in a disjoint-set implementation using both union by rank and path compression, any sequence of m MAKESET, FIND and UNION operations takes only $O(m)$ time if all the UNION operations appear before any of the FIND operations.

Homework 4 (5 Points)

Show that there always exists a sequence of at most m augmenting paths that compute the maximum flow in a network of m edges.

Tutorial Exercise 1

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices, as shown in the figures below. We denote the vertex in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1$, $i = \text{n}$, $j = 1$, or $j = \text{n}$. Given $m \leq n^2$ starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ in the grid, the escape problem is to determine whether or not there are m vertex-disjoint paths from the starting points to any m different points on the boundary. For example, the first grid below has an escape, but the second grid does not. Starting points are shown in black.



Describe an efficient algorithm to solve the escape problem, and analyze its running time. **Hint:** You might need capacities at both nodes and edges. Argue that this is not a problem.

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In any such war in Europe the rail
network of Eastern Europe would be
an important target.  It therefore
appears reasonable to illustrate the
method [of using a flow network] by
applying it to the Eastern European
rail net.
                    - T.E. Harris, F.S. Ross
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