Prof. Dr. Susanne Albers
Dr. Suzanne van der Ster
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Fakultät für Informatik
Technische Universität München

Problem set 10
June 17, 2016
Summer Semester 2016

Online and Approximation Algorithms

Due June 24, 2016 before 10:00

Exercise 1 (Minimum cost perfect matching - 10 points)

Let $G = (V, E)$ be a graph with edge costs satisfying the triangle inequality, and $V' \subseteq V$ be a set of even cardinality (and let $|V|$ be even as well).
Prove or disprove: The cost of a minimum cost perfect matching on V' is bounded from above by the cost of a minimum cost perfect matching on V.

Exercise 2 (1,2-TSP - 10 points)

Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Note that G clearly satisfies the triangle inequality.
Give a 4/3-approximation algorithm for TSP on this special class of graphs.
Hint: Start by finding a minimum 2-matching in G. A 2-matching is a subset S of edges such that every vertex is incident to exactly 2 edges of S.

Exercise 3 (Sorted List Scheduling - 10 points)

In the lecture, it was shown that the Sorted List Scheduling algorithm achieves an approximation ratio of $\frac{4}{3}$ for the problem of makespan minimization. Show that this factor is tight for $m \to \infty$.

Exercise 4 (Makespan Minimization - 10 points)

We consider the problem of scheduling n jobs with processing times $p_1, p_2, \ldots, p_n$ on m machines, where the goal is to minimize the makespan.
Consider the algorithm which starts from an arbitrary schedule σ and modifies the schedule iteratively as follows. It identifies a job j currently assigned to machine p and it moves j to machine q if the new completion time of q after the move is smaller than the initial completion time of machine p. The algorithm terminates if it is not possible to perform such a move for any job.

(a) Show that this algorithm terminates.

(b) Show that this algorithm has an approximation ratio of 2.