Prof. Dr. Susanne Albers
Dr. Suzanne van der Ster
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Fakultät für Informatik
Technische Universität München

Problem set 3
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Summer Semester 2016

# Online and Approximation Algorithms

*Due May 6, 2016 before 10:00*

## Exercise 1 (RMARK - 10 points)

Consider a memory system with fast memeory of size $k$. RMARK is the randomized online paging algorithm that works as follows.
Initially, all pages are unmarked. Whenever a page is requested it becomes marked. When a page is brought into fast memory, it replaces a randomly and uniformly chosen page from the set of unmarked pages that are in fast memory. When all pages in fast memory are marked and a page fault occurs, all pages become unmarked. Prove that RMARK is $H_k$-competitive against oblivious adversaries when the total number of pages equals $k+1$.

## Exercise 2 (RMARK II - 10 points)

Show an example with corresponding request sequence such that the cost of RMARK (with specific random choices) is greater than $H_k$ times the cost of the optimal offline algorithm.
*Hint: An example exists consisting of only 4 pages and a fast memory of size 2.*

## Exercise 3 (Ski Rental, Yao's Principle - 10 points)

Use Yao's Minimax Principle to prove that the competitive ratio of any randomized algorithm for the ski rental problem is lower bounded by $\frac{7}{6}$.

## Exercise 4 (String Scanning, Yao's Principle - 10 points)

Consider the following problem. Given a string $x \in \{0,1\}^n$, we want to determine if $x$ contains two consecutive 1.
By using Yao's Minimax Principle, show that the expected number of bits inspected by any randomized algorithm is $\Omega(n)$.